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Junior College 2
The Ultimate Exam Revision Notes

H2 Mathematics

Structured Revision (Functions)

- Inverse and Self-inverse Functions
- Composite Functions
- Piecewise Functions

Ready to level up your JC H2 Math? This set of questions helps you reflect on your strengths and uncover opportunities for deeper learning.

Remember to try and attempt all questions before checking the answers!

Every solution starts with a step — begin your journey to JC Math mastery today.

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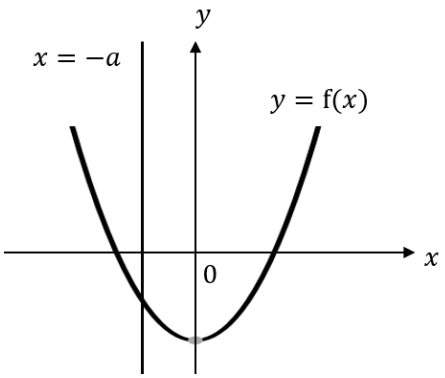
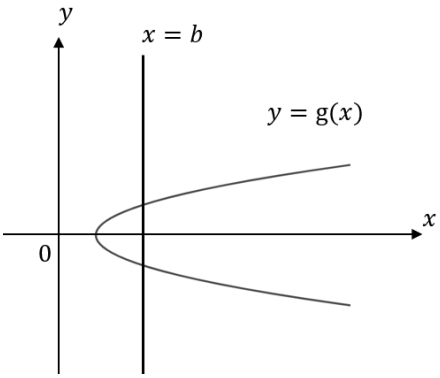
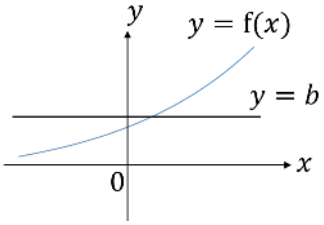
Concept Summary

To show a function	
 The graph $\mathrm{f}(x)$ is a function, since the vertical line $x = -a$ cuts the graph at most once.	 The graph $\mathrm{g}(x)$ is **NOT** a function, since the vertical line $x = b$ cuts the graph at two distinct points.

To show that a function is one-one	
Method 1: Horizontal Line Test	**Method 2: Show that function is strictly increasing/decreasing.**
 Since the horizontal line $y = b$ cuts the graph at most once, $y = \mathrm{f}(x)$ is one-one function.	The function $\mathrm{f}(x)$ is defined by $\mathrm{f} : x \mapsto \ln(2x + 5) + 5x + e^x, \qquad x > -\frac{5}{2}$ Since $\mathrm{f}'(x) = \frac{2}{2x+5} + 5 + e^x > 0$ for $x > -\frac{5}{2}$, the function is strictly increasing. ∴ The function is one-one.

Finding the range of a function	
One-one function	**Not one-one**
Substitute the end points of the domain	Use GC

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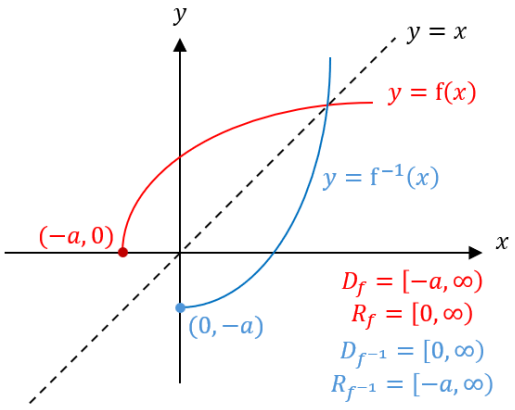
Inverse functions



Domain and Range

1) $D_{f^{-1}} = R_f$

2) $R_{f^{-1}} = D_f$

3) $ff^{-1}(x) = f^{-1}f(x)$
[Remember to check domain!]

Relationship

$f(x)$ and $f^{-1}(x)$ are reflection about the line $y = x$.

$f(x)$	$f^{-1}(x)$
Coordinate: (a, b)	Coordinate: (b, a)
Asymptote: $y = k$	Asymptote: $x = k$
Asymptote: $x = h$	Asymptote: $y = h$

Finding inverse function

Given

$\mathrm{f} : x \mapsto x^2 + 4x + 9, x \in \mathbb{R}, x \leq -2,$

find f^{-1} in similar form.

Step 1: Let $y = \mathrm{f}(x)$ and make x the subject

$y = x^2 + 4x + 9 = (x + 2)^2 + 5$

$(x + 2)^2 = y - 5$

$x = -2 \pm \sqrt{y - 5}$

Step 2: Check domain and determine +/-

Since $x \leq -2$, $x = -2 - \sqrt{y - 5}$

Step 3: Find domain of f^{-1}, $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}}$.

Range of f: $y \geq 5$

Step 4: LHS – replace x with $\mathrm{f}^{-1}(x)$, RHS – replace y with x, define in similar form.

$\mathrm{f}^{-1} : x \mapsto -2 - \sqrt{x - 5}, x \in \mathbb{R}, x \geq 5$

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Restricting domain so that inverse function exists

If a function is not one-one, restrict the domain of the original function.

- If f(x) is quadratic, do completing the squares first then restrict from line of symmetry.
- If f(x) is complicated, use graphic calculator.

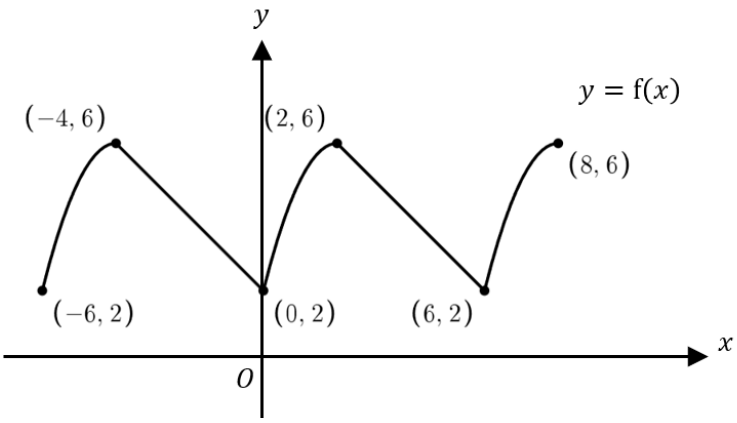
Example, for $y = x^2 + 1$, we restrict the domain to $x \geq 0$ to have one-one function.

Piecewise & Periodic functions

The function f is defined by

$$\mathrm{f}(x) = \begin{cases} 6 - (x-2)^2, & 0 \leq x \leq 2 \\ 8 - x, & 2 < x \leq 6 \end{cases}$$

Given that $\mathrm{f}(x) = \mathrm{f}(x+6)$, sketch $\mathrm{f}(x)$ for $-6 \leq x \leq 8$.



Sketching $\mathrm{f}(x)$	**Evaluate**
Step 1: Start from principal domain $0 \leq x \leq 6$ **Step 2:** Repeat for given domain (For periodic function)	Find $\mathrm{f}(13)$ $\mathrm{f}(13) = \mathrm{f}(1) = 6 - (1-2)^2$ $\mathrm{f}(13) = 5$

Area

$$\int_{-6}^{8} \mathrm{f}(x)\, dx$$

$= 3 \times$ area under curve $+ 2$ trapezium

$$= 3\int_{0}^{2} 6 - (x-2)^2\, dx + 2\left(\frac{1}{2}\right)(6+2)(4)$$

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Composite functions

$$\mathrm{fg}(x) = \mathrm{f}[\mathrm{g}(x)]$$

1) **For composite function to exist, range of inner function ⊆ domain of outer function**

   For $\mathrm{fg}(x)$ to exist, $\mathrm{R_g} \subseteq \mathrm{D_f}$.

2) **Domain of composite function = Domain of inner function**

   $\mathrm{D_{fg}} = \mathrm{D_g}$

3) **Range of composite function**

   Step 1: Find range of inner function

   Step 2: Sketch graph of outer function (may need GC)

   Step 3: Deduce range of composite function

In general, $\mathbf{R_{fg} \neq R_f}$

$$\mathbf{R_{fg} = R_f \text{ if } R_g = D_f}$$

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Checklist

For **inverse functions**, I know how to:

- ☐ Determine a function has an inverse by using horizontal line test.
- ☐ Restrict the domain such that a function becomes one-one.
- ☐ Find the inverse function $y = \mathrm{f}^{-1}(x)$ algebraically.
- ☐ Sketch the graph of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ over the symmetry line $y = x$.
- ☐ Sketch identity functions such as $y = \mathrm{ff}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$.

For **composite functions**, I know how to

- ☐ Determine whether composite functions $\mathrm{fg}(x)$ exist by showing the range of inner function g is a subset of domain of outer function f.
- ☐ Find $\mathrm{fg}(x)$ by substituting $\mathrm{g}(x)$ into $\mathrm{f}(x)$.
- ☐ Find the range of composite function $\mathrm{fg}(x)$ by analysing the range of inner function $\mathrm{g}(x)$ and the sketch out outer function $\mathrm{f}(x)$.

For **self-inverse functions**, I know how to

- ☐ Verify $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ by showing $\mathrm{ff}(x) = x$.
- ☐ Find $\mathrm{f}^3(x)$, $\mathrm{f}^{100}(x)$, and other large power n for $\mathrm{f}^n(x)$, using $\mathrm{f}^2(x) = x$, $\mathrm{f}^4(x) = \mathrm{f}^2(x) = x$, etc.

For **piecewise and periodic functions**, I know how to

- ☐ Sketch the graph of piecewise function using the principal domain.
- ☐ Sketch the graph of periodic piecewise function using $\mathrm{f}(x) = \mathrm{f}(x + a)$
- ☐ Find the area of region for periodic piecewise functions.

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Exam Grade Questions

Practice 1 **[Level: Basic]**

(a) The function f is defined by $\mathrm{f} : x \mapsto 2 + \frac{3}{x}, x \in \mathbb{R}, x > 0$.

(i) Sketch the graph of $y = \mathrm{f}(x)$. Hence, show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) On the same diagram as in part (i), sketch the graphs of $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$. [2]

(iv) Explain why f^2 exists and find $\mathrm{f}^2(x)$. [2]

(b) The function h is defined by $\mathrm{h} : x \mapsto \frac{3-x}{x^2-1}, x \in \mathbb{R}, x \neq \pm 1$. Find algebraically the range of h, giving your answer in exact form. [4]

Concepts	Possible Method(s):
Check if a function is one-one	• Horizontal line test • Strictly increasing/decreasing by finding $\mathrm{f}'(x)$.
Check if a function is self-inverse	• Find $\mathrm{f}^{-1}(x)$ and check if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. • Verify if $\mathrm{ff}(x) = x$.
Find the domain of inverse function	Domain of f^{-1} = Range of f. Sketch $f(x)$ to find range.
Check if composite function gf exists	• Check if range of inner function is a subset of domain of outer function (i.e. For $\mathrm{gf}(x)$ to exist, $\mathrm{R_f} \subseteq \mathrm{D_g}$)
Find the range of a composite function gf	State the range of inner function f, sketch the outer function g, analyse the range of the graph using range of inner function as domain.

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Practice 2 **[Level: Intermediate]**

The functions f and g are defined by

$$\mathrm{f} : x \mapsto a + \frac{2}{3(x-a)} \qquad \text{for } x \in \mathbb{R}, x \neq a.$$

$$\mathrm{g} : x \mapsto \frac{a}{2} - \left(x - \sqrt{\frac{a}{2}+1}\right)^2 \qquad \text{for } x \geq 0.$$

where a is a positive constant.

(a) Define, in a similar form, the inverse function f^{-1} and show that $\mathrm{f}^2(x) = x$. [3]

(b) Hence, find $\mathrm{f}^{2k+1}(2a)$ for positive integer k, giving your answer in terms of a. [2]

(c) Show that the composite function fg exists and find the range of fg. [3]

Concepts	Possible Method(s):
Find the rule of inverse function $\mathrm{f}^{-1}(x)$	Let $y = \mathrm{f}(x)$ and make x the subject. Then interchange x and y to get $\mathrm{f}^{-1}(x)$
State the domain of inverse function f^{-1}	Domain of f^{-1} = Range of f. Sketch $y = \mathrm{f}(x)$ to find range.
Use properties of self-inverse functions	$\mathrm{f}(x) = \mathrm{f}^{-1}(x) \Rightarrow \mathrm{f}^2(x) = x$
Check if composite function fg exists	Check if range of inner function is a subset of domain of outer function (i.e. For $\mathrm{fg}(x)$ to exist, $\mathrm{R_g} \subseteq \mathrm{D_f}$)
Find the range of a composite function gf	State the range of inner function f, sketch the outer function g, analyse the range of the graph using range of inner function as domain.

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Practice 3 **[Level: Intermediate]**

The function f is defined by

$$\mathrm{f}: x \mapsto \begin{cases} 1-|3x-1| & \text{for } 0 \le x \le 2, \\ (x-2)^2-4 & \text{for } 2 < x \le 4. \end{cases}$$

It is given that $\mathrm{f}(x+4) = \mathrm{f}(x)$ for all real values of x.

(a) Sketch the graph of $y = \mathrm{f}(x)$ for $-1 \le x \le 9$, stating the coordinates of the end points. Find the range of f. [4]

(b) Find [3]

$$\int_0^8 \mathrm{f}(x) + 4\,dx$$

Concepts	Possible Method(s):
Graph of piecewise function	Sketch the graph for the principle domain first $0 \le x \le 4$.
Identify periodic function $\mathrm{f}(x) = \mathrm{f}(x+4)$	$\mathrm{f}(x) = \mathrm{f}(x+4)$ implies that the graph is repeating every 4 units, period is 4.
Evaluating definite integral in piecewise function	Check the region that gives the definite integral. Decide if you need to move the graph (up by 4 units in this case) or evaluate separately $\int_0^8 \mathrm{f}(x)\,dx + \int_0^8 4\,dx$

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Practice 4 **[Level: Advanced]**

The functions f and g are defined by:

$$\mathrm{f}: x \mapsto |ax - x^2|, \qquad x \in \mathbb{R}$$
$$\mathrm{g}: x \mapsto ae^{-x} + 1, \qquad x \geq 0$$

where a is a positive constant and $a \geq 5$.

(a) Sketch the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{f}(x)$ on separate diagrams.

Hence find the range of the composite function fg, leaving your answer in exact form in terms of a. [5]

(b) Given that the domain of f is restricted to the subset of $\mathbb{R}$ for which $x \geq k$, find the smallest value of k, in terms of a, for which f^{-1} exists. Hence, without finding the expression for $\mathrm{f}^{-1}(x)$, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on a single diagram, stating the exact coordinates of any points of intersection with the axes. [3]

(c) With the restricted domain of f in part (b), find with clear working, the set of values of x such that $\mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$. [2]

Concepts	Possible Method(s):
Check if composite function gf exists	• Check if range of inner function is a subset of domain of outer function (i.e. For $\mathrm{gf}(x)$ to exist, $\mathrm{R_f} \subseteq \mathrm{D_g}$)
Find the range of a composite function gf	State the range of inner function f, sketch the outer function g, analyse the range of the graph using range of inner function as domain.
Check if a function is one-one	• Horizontal line test • Strictly increasing/decreasing by finding $\mathrm{f}'(x)$.
Sketch the graph of inverse function	Reflect the restricted $\mathrm{f}(x)$ across the line $y = x$.
Find the solution to identify functions $\mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$	Domain of composite function = Domain of inner function. Solution is at Domain of f^{-1} intersecting Domain of f (i.e. $\mathrm{D}_{\mathrm{f}^{-1}} \cap \mathrm{D}_{\mathrm{f}}$)

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Quiz

Q1. **[Time: 18 minutes]**

The function f is defined by $\mathrm{f} : x \mapsto \frac{2x+6}{x-2},\ x \in \mathbb{R}, x > 2.$

(a) Find f^{-1} in a similar form. [3]

(b) Find the solution set for $\mathrm{f}^2(x) = x$. [1]

It is given that k is a constant such that $k > 3$.

The function g is defined by $\mathrm{g} : x \mapsto (x-5)^2 + k, x \in \mathbb{R}, x > 2$.

(c) Show that fg exists. [1]

(d) Find the range of values of fg, in terms of k. [2]

The function ϕ is defined by $\phi : x \mapsto \frac{2x+a}{x-2}$, where a is a constant, $x \in \mathbb{R}, x > 2$.

(e) Given that ϕ^{-1} exists, state the value that a cannot take, justifying your answer. [2]

A function h is said to be self-inverse if $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$ for all x in the domain of h.

(f) State the range of values of a such that ϕ is a self-inverse function. [1]

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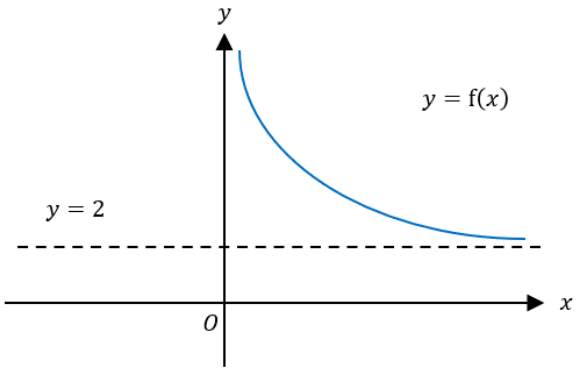
Exam Grade Questions (Solutions)

Practice 1 **[Level: Basic]**

(ai)



Since any horizontal line $y = k, k \in \mathbb{R}$ intersects the graph of f at most once, f is one-one. ∴ f has an inverse.

(aii)

$y = 2 + \frac{3}{x}$

$\frac{3}{x} = y - 2$

$x = \frac{3}{y-2}$

$\therefore \mathrm{f}^{-1}(x) = \frac{3}{x-2}$

$D_{f^{-1}} = R_f = (2, \infty)$

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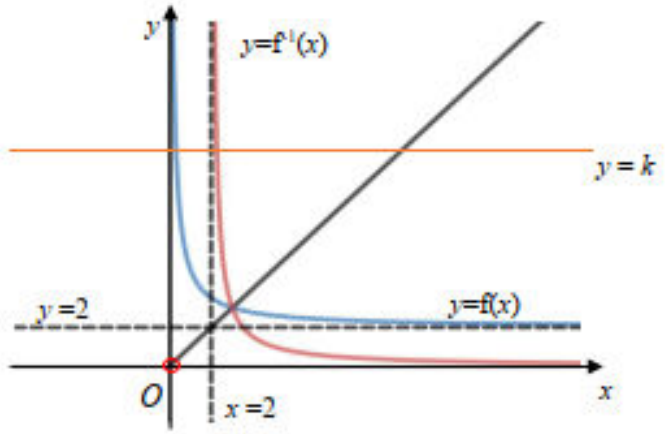
(aiii)



(iv)

$D_f = (0, \infty), R_f = (2, \infty)$

Since $R_f \subseteq D_f$, f^2 exists.

(b)

Let $y = \frac{3-x}{x^2-1}$

$x^2y - y = 3 - x$

$(y)x^2 + (1)x + (-y - 3) = 0$

To find range of y such that the graph exists, we want equation to have real roots.

$b^2 - 4ac \geq 0$

$1^2 - 4(y)(-y - 3) \geq 0$

$1 + 4y^2 + 12y \geq 0$

To solve for exact range of y, either use quadratic formula or completing the squares.

$4(y^2 + 3y) + 1 \geq 0$

$4\left[\left(y + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 1 \geq 0$

$4\left(y + \frac{3}{2}\right)^2 - 8 \geq 0$

$\left(y + \frac{3}{2}\right)^2 - \left(\sqrt{2}\right)^2 \geq 0$

$\left(y + \frac{3}{2} - \sqrt{2}\right)\left(y + \frac{3}{2} + \sqrt{2}\right) \geq 0$

$\therefore y \leq -\frac{3}{2} - \sqrt{2}$ or $y \geq -\frac{3}{2} + \sqrt{2}$

$R_h = \left(-\infty, -\frac{3}{2} - \sqrt{2}\right] \cup \left[-\frac{3}{2} + \sqrt{2}, \infty\right)$

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Practice 2 **[Level: Intermediate]**

(a) Let $y = a + \frac{2}{3(x-a)}$

$y - a = \frac{2}{3(x-a)}$

$x - a = \frac{2}{3(y-a)}$

$x = a + \frac{2}{3(y-a)}$

$D_{f^{-1}} = R_f = \mathbb{R}\backslash\{a\}$

$f^{-1}: x \mapsto a + \frac{2}{3(x-a)}, x \in \mathbb{R}, x \neq a,$

Method 1

Since $f^{-1} = f, f^2(x) = ff^{-1}(x) = x$

Method 2

$f^2(x) = f\left(a + \frac{2}{3(x-a)}\right)$

$= a + \frac{2}{3\left(a+\frac{2}{3(x-a)}-a\right)}$

$= a + \frac{2}{3\left(\frac{2}{3(x-a)}\right)}$

$= a + (x - a)$

$= x$

(b) $f^{2k+1}(x) = ff^{2k}(x) = f(x)$

$f^{2k+1}(2a) = f(2a) = a + \frac{2}{3a}$

(c) For maximum value of $g(x) = \frac{a}{2} - \left(x - \sqrt{\frac{a}{2}+1}\right)^2$,

It occurs at $x = \sqrt{\frac{a}{2}+1}$ [when $\left(x - \sqrt{\frac{a}{2}+1}\right)^2 = 0$]

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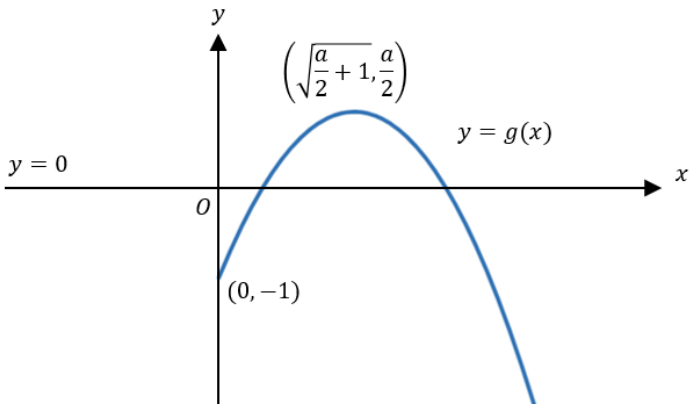
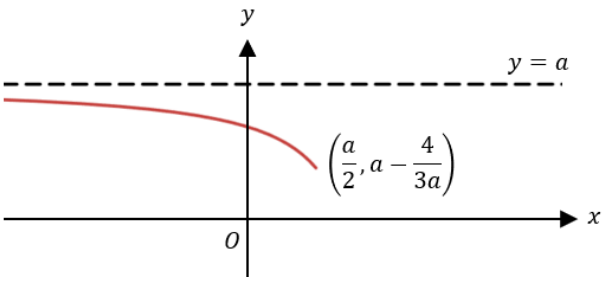


From the sketch, $R_g = \left(-\infty, \frac{a}{2}\right] \subset \mathbb{R}\backslash\{a\} = D_f$

Therefore, fg exists.

When we restrict $D_f = \left(-\infty, \frac{a}{2}\right]$



Range of fg $= \left[a - \frac{4}{3a}, a\right)$

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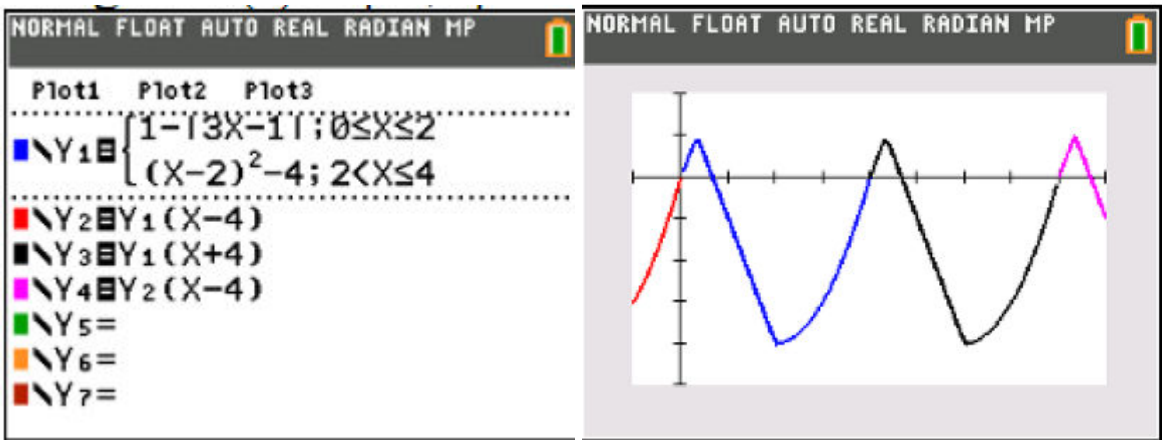
Practice 3 **[Level: Intermediate]**

Solution

(a)



[Piecewise function: Press math then option B]

[Y_1, Y_2: Press alpha trace]

$\mathrm{f}(9) = \mathrm{f}(5) = \mathrm{f}(1) = 1 - |3 - 1| = -1$

$\mathrm{f}(-1) = f(3) = (3-2)^2 - 4 = -3$

Coordinates of end-points $(-1, -3)$ and $(9, -1)$

$\mathrm{R_f} = [-4, 1]$

(b)

The graph is shifted up by 4 units.

From $x = 0$ to $x = \frac{1}{3}$, area of trapezium $= \frac{1}{2}(4+5)\left(\frac{1}{3}\right) = \frac{3}{2}$

From $x = \frac{1}{3}$ to $x = 2$, area of triangle $= \frac{1}{2}\left(2 - \frac{1}{3}\right)(5) = \frac{25}{6}$

From $x = 2$ to $x = 4$, Area under curve $= \int_4^2 (x-2)^2\,dx = \left[\frac{(x-2)^3}{3}\right]_4^2 = \frac{8}{3}$

$$\int_0^8 \mathrm{f}(x)\,dx = 2\left(\frac{3}{2} + \frac{25}{6} + \frac{8}{3}\right) = \frac{50}{3}$$

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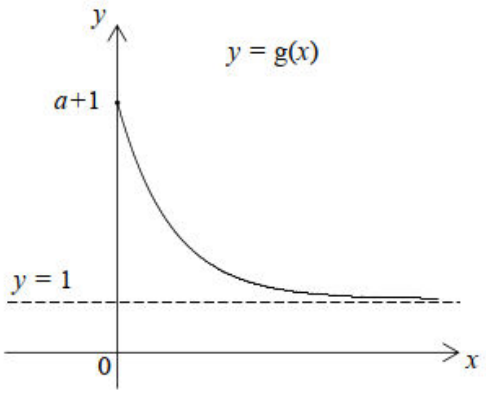
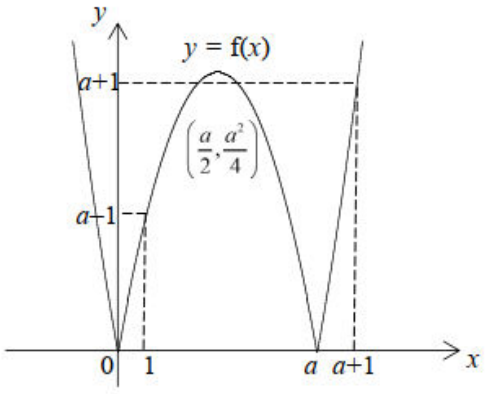
Practice 4 **[Level: Advanced]**

(a)





From graph, $R_g = (1, a+1]$

For $a \geq 5$, we have the following

$\mathrm{f}(1) = |a-1| = a-1$

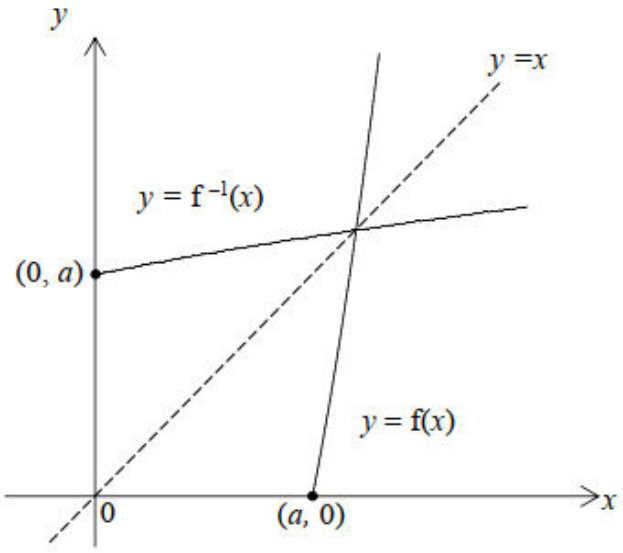
$\mathrm{f}(a+1) = |a(a+1) - (a+1)^2| = |a^2 + a - a^2 - 2a - 1| = |-a-1| = a+1$

$\frac{a^2}{4} \geq \frac{25}{4} > a+1 \quad \Rightarrow$ The right endpoint is below maximum point.

$\therefore R_{fg} = \left[0, \frac{a^2}{4}\right]$

(b)

For f^{-1} to exist, $k = a$ (i.e. $\mathrm{f}(x)$ is one-one for $x \geq a$)



(c)

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For $\mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$, the solution is at $\mathrm{D}_{\mathrm{f}^{-1}} \cap \mathrm{D}_{\mathrm{f}}$

$$[0, \infty) \cap [a, \infty) = [a, \infty)$$

$\therefore$ The solution set of values of x is $[a, \infty)$

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Quiz

Q1. **[Time: 18 minutes]**

Solution

(a)

$$y = \frac{2x+6}{x-2}$$
$$xy - 2y = 2x + 6$$
$$x(y-2) = 2y + 6$$
$$x = \frac{2y+6}{x-2}$$
$$\mathrm{f}^{-1}(x) = \frac{2x+6}{x-2}, \mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = (2, \infty)$$
$$\therefore \mathrm{f}^{-1} : x \mapsto \frac{2x+6}{x-2}, \qquad x > 2$$

(b)

Since $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$,

$\mathrm{f}^2(x) = x \Rightarrow \mathrm{f}(x) = \mathrm{f}^{-1}(x)$

Solution is at $\mathrm{D}_{\mathrm{f}} \cap \mathrm{D}_{\mathrm{f}^{-1}}$

$(2, \infty) \cap (2, \infty) = (2, \infty)$

$\therefore$ The solution is $\{ x \in \mathbb{R} : x > 2 \}$

(c)

$$R_g = [k, \infty)$$
$$D_f = (2, \infty)$$

Since $R_g \subseteq D_f$, as $k > 3 > 2$,

fg exists.

(d)

$$\mathrm{g}(x) = (x-5)^2 + k, x > 2$$

[Sketch $\mathrm{g}(x)$, turning point at $(5, k)$, $\mathrm{R}_{\mathrm{g}} = [k, \infty)$]

$$(2, \infty) \xrightarrow{g} [k, \infty) \xrightarrow{f} \left(2, \frac{2k-6}{k-2}\right]$$
$$\therefore \mathrm{R}_{\mathrm{fg}} = \left(2, \frac{2k-6}{k-2}\right]$$

(e)

$$y = \frac{2x+a}{x-2} = \frac{2(x-2)+4+a}{x-2} = 2 + \frac{4+a}{x-2}$$

For ϕ^{-1} to exist, ϕ must be one-one, $4 + a \neq 0$, otherwise $y = 2$ is not one-one.

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$\therefore a \neq -4$

(f)

Using the result from (a) and (e), ϕ is a self-inverse function for any $a \neq -4$,

$\therefore a < -4$ or $a > -4$.

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