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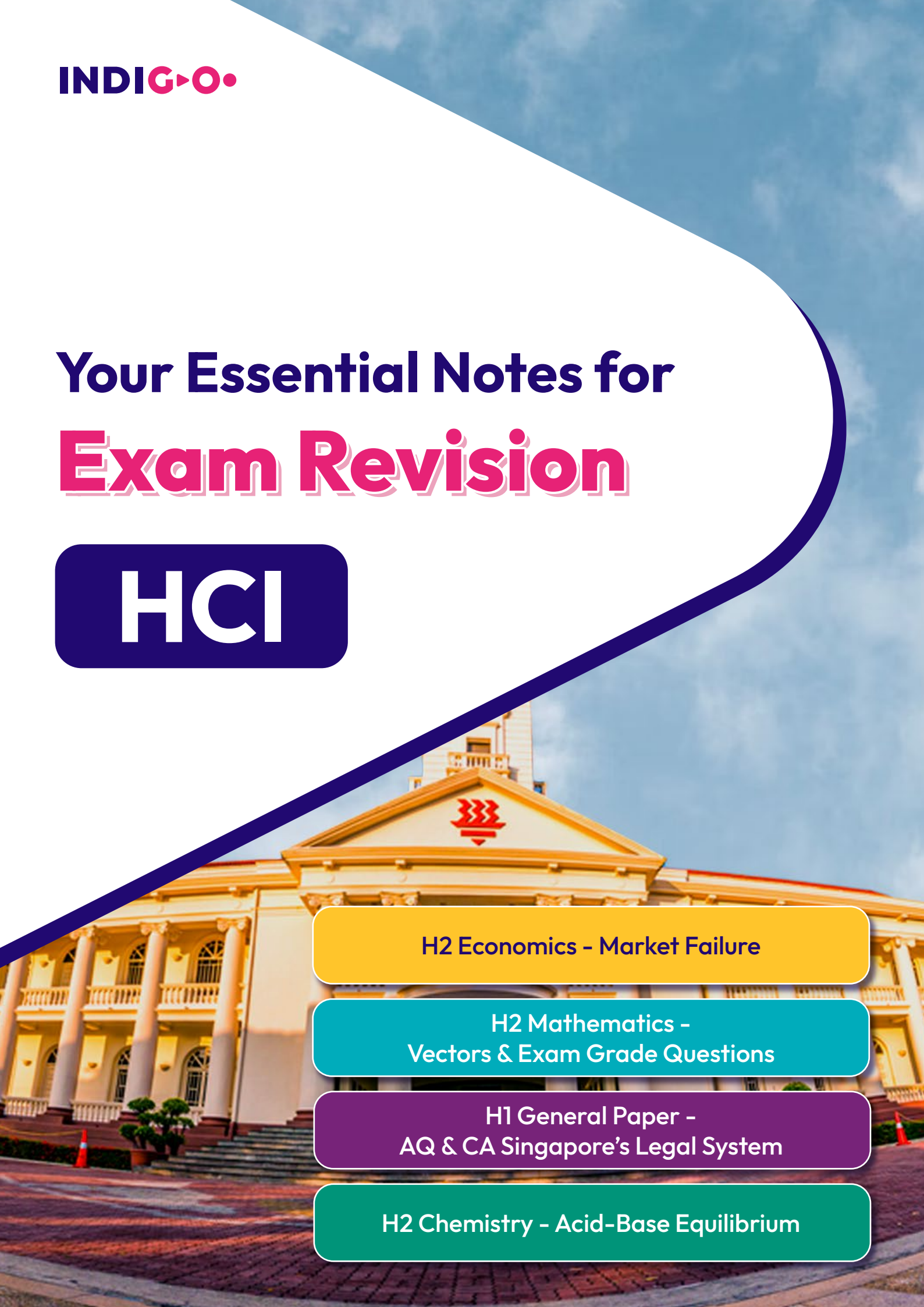
Your Essential Notes for Exam Revision

HCI



H2 Economics - Market Failure

H2 Mathematics -
Vectors & Exam Grade Questions

H1 General Paper -
AQ & CA Singapore's Legal System

H2 Chemistry - Acid-Base Equilibrium

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Microeconomic Objectives and Policies

Integrative Application: Land Transport in Singapore

Market failure: negative externality in consumption. Third parties (other road users and passers-by) bear congestion and pollution costs from an additional car trip.

The failure, in the standard four moves

1. Self-interested individual weighs only the costs and benefits accruing to itself. MPB of an additional car trip is the driver's additional utility; MPC is the additional fuel and running cost. The driver travels where MPB = MPC, at Qm.
2. MEC is borne by third parties, not by the individual (no compensation paid). MEC of an additional trip is the additional congestion delay and healthcare cost imposed on others, which is unpriced and excluded from MPC.
3. MEC drives MSC above MPC (MSC = MPC + MEC), so the social optimum Qs, where MSB = MSC, lies below Qm.
4. At Qm, the market over-consumes; for units between Qs and Qm, MSC > MSB, indicating that the additional cost imposed on society is greater than the additional benefit gained. Since each of these units creates a net welfare loss equal to the vertical gap between the MSC and MSB curves, the cumulative welfare loss is represented by the deadweight loss, area abc.

Policy mix and evaluation

Policy	Mechanism	Key limitation
Electronic Road Pricing (market-based)	Charges drivers a price = MEC at Qs, varying by road and time to match local MEC, so drivers internalise the external cost and usage falls toward Qs.	Requires accurate MEC data; effectiveness depends on PED; may divert traffic to un-priced roads.
Improving public transport (moral suasion / substitution)	Better rail and bus networks raise the substitutability of public for private transport, shifting demand off the roads.	Effectiveness depends on how close a substitute public transport is; car ownership as a status symbol weakens the effect; new lines have a long time-lag.
Vehicle Quota System / COE (hybrid)	The regulator releases a set number of certificates; a bidding market prices them, allocating car ownership to those who value it most.	Targets ownership, not usage, so congestion can persist; the sunk-cost effect dulls sensitivity to ERP; higher-income drivers outbid lower-income ones (equity concern).

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This case shows the topic's central lesson in one market: a single failure is best addressed by a calibrated policy mix, and each instrument must be evaluated against information, elasticity, cost and equity — with the efficiency–equity trade-off (Chain 4.3) made explicit through the COE's distributional effect.

Policies to Correct Market Failure

A policy is only as good as its match to the underlying failure. The table below maps each failure to its principal remedies; the sections that follow give the transmission mechanism (as a chain) and the limitations of each.

Source of failure	Principal policies
Negative externality	Tax = MEC; tradable permits; quotas / bans (command and control).
Positive externality	Subsidy = MEB; direct or joint provision; public education.
Information failure (merit / demerit)	Public education; regulation (labelling, age limits, minimum standards).
Asymmetric information	Regulation; screening and signalling; universal (compulsory) provision.
Public good	Direct provision funded by taxation.

3.1 Taxation (for negative externalities)

A per-unit (specific) tax set equal to the MEC at Qs raises the firm's effective marginal private cost so that it internalises the external cost.

Chain 3.1 — Unit tax = MEC at Qs level (reproduce this):

Tax = MEC raises MPC to MPC1 (= MSC) upwards by the per-unit tax imposed → new private equilibrium at Qs

→ MPB = MSC at Qs → deadweight loss (triangle abc) eliminated → allocative efficiency restored

Limit: if measured MEC ≠ true MEC → tax ≠ MEC → Q ≠ Qs → residual DWL → government failure

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How to draw — Tax to correct a negative externality	
Steps **1. Starting point:** The negative-externality diagram, with MSC = MPC + MEC above MPC, over-production at Qm and DWL triangle abc. **2. Policy shift:** Impose a specific tax = MEC. MPC shifts up by the tax to MPC1 = MPC + tax, which coincides with MSC. **3. New equilibrium:** The firm now produces where MPB = MPC1, i.e. at Qs (the old Qm falls to Qs). Label this Qp1 = Qs. **4. Result:** MPB = MSC at Qs; the deadweight loss triangle abc is eliminated and allocative efficiency is restored.	*Diagram space — draw and label here*

Advantages	Limitations
Flexible — the tax can be adjusted to the size of the externality. Targeted — discourages the specific harmful activity directly. Raises revenue that can fund complementary measures.	Imperfect information — the intangible MEC is hard to quantify, so the government may over-tax (causing under-production) or under-tax (over-production persists). Effectiveness depends on PED: if demand is price-inelastic, a large tax cuts quantity only a little. May worsen equity — the tax is regressive if it falls more heavily on lower-income consumers.

3.2 Subsidy (for positive externalities)

Precision note: the subsidy makes agents internalise the external benefit. It works on the benefit-side divergence (MSB = MPB + MEB), reducing the effective cost so that private output expands to Qs.

Chain 3.2 — Per-unit indirect subsidy = MEB at Qs level (reproduce this):

Indirect subsidy = MEB reduces effective MPC downwards by the per-unit subsidy provided → new private equilibrium at Qs

→ MSB = MPC at Qs → deadweight loss (triangle abc) eliminated → allocative efficiency restored

Limit: if subsidy ≠ MEB → over- or under-correction → Q ≠ Qs → government failure

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How to draw — Subsidy to correct a positive externality	
Steps **1. Starting point:** The positive-externality diagram, with MSB = MPB + MEB above MPB, under-production at Qm and DWL triangle abc. **2. Policy shift:** Grant a specific subsidy = MEB. This lowers effective MPC, shifting supply (MPC) down/right to MPC1. **3. New equilibrium:** The consumer now transacts where MSB (via the subsidised price) supports output at Qs (Qm rises to Qs). Label Qp1 = Qs. **4. Result:** MSB = MPC at Qs; the deadweight loss triangle abc is eliminated and allocative efficiency is restored.	*Diagram space — draw and label here*

Advantages	Limitations
Flexible — the subsidy can be scaled to the size of the external benefit. Targeted — encourages the specific beneficial activity.	Imperfect information — the MEB is hard to quantify, risking over- or under-subsidy. Effectiveness depends on PED for the good. Opportunity cost — the subsidy must be funded, possibly through higher taxes elsewhere. May breed inefficiency — guaranteed revenue can raise X-inefficiency and weaken the incentive to innovate (dynamic inefficiency).

3.3 Tradable Permits (hybrid: command-and-control + market)

The government caps total emissions at the socially optimal level by issuing that many permits, then lets firms trade them. The cap fixes the quantity; the market sets the permit price.

1. Total permits issued = the emissions consistent with Qs. Each permit entitles the holder to a set quantity of emissions; exceeding one's permits incurs heavy sanctions.
2. A market price for permits emerges from firms' demand to pollute. A firm that can cut emissions cheaply (low abatement cost) does so and sells its spare permits; a firm that finds abatement costly buys permits instead.
3. Abatement is thus concentrated in the lowest-cost firms, so the target is met at the minimum total cost to society, while total output settles at Qs where MSB = MSC.

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Chain 3.3 — Tradable permits (reproduce this):
Cap total permits at Qs → permit market price emerges → low-abatement-cost firms abate and sell permits
→ target met at least cost, total output = Qs → MSB = MSC → allocative efficiency at minimum cost

Advantages	Limitations
Certain quantity outcome — the cap fixes total emissions, unlike a tax. Cost-effective — abatement is done by the firms that can do it most cheaply. Incentivises adoption of cleaner technology to free up permits to sell.	Imperfect information — the efficient cap (and hence Qs) is hard to determine. Distributional disputes over the initial allocation of permits. High monitoring and enforcement costs. Weaker incentive for long-term green investment if the payoff is uncertain.

3.4 Quotas and Bans (command and control)

Quotas

- A legal limit on quantity, set at the socially optimal output Qs.

Advantages	Limitations
Greater certainty of hitting the target output than a tax. Targeted at the specific activity.	Imperfect information — Qs is hard to determine. Blunt — does not achieve the reduction at least cost across firms. Displaces the price mechanism — output is no longer set by the market. Costly to enforce and monitor.

Bans

Used where the MEC is very large (where MSC exceeds MSB at **all** output levels). A ban sets output to zero, which is welfare-improving only if the deadweight loss it creates is smaller than the deadweight loss it removes.

Case 1 — ban improves welfare	Case 2 — ban worsens welfare
DWL created by the ban (triangle dae, from lost beneficial consumption) < DWL removed (triangle abc, from over-consumption). Net welfare gain: society is better off after the ban.	DWL created by the ban (triangle dae) > DWL removed (triangle abc). Net welfare loss: society is worse off after the ban.
Diagram space — draw and label here	*Diagram space — draw and label here*

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Case 1 — ban improves welfare	Case 2 — ban worsens welfare

3.5 Public Education and Moral Suasion

- **Public education** targets *information failure*: accurate information corrects perceived valuations.
- **Moral suasion** targets *externalities*: it appeals to civic and ethical responsibility to change behaviour. The two are not the same and address different failures.

Chain 3.4 — Public education for information failure (reproduce this):
Accurate information provided → perceived MPB (or MPC) adjusts toward the actual value → quantity shifts toward Qs
→ deadweight loss reduced → allocative efficiency improved

Advantages	Limitations
Tackles the root cause of information failure rather than the symptom. No distortionary tax or displacement of the price mechanism.	Uncertain outcome — it cannot be enforced. Long gestation — mindsets change slowly. Low effectiveness against addictive goods (alcohol, cigarettes, gambling) and where habits are entrenched.

3.6 Regulation (for information failure and asymmetric information)

- Mandatory disclosure — e.g. sugar and sodium labelling, health warnings on cigarette packs — narrows the gap between perceived and actual private cost or benefit.
- Protective law — e.g. the Lemon Law and the Consumer Protection (Fair Trading) Act — reduces the buyer's fear of hidden low quality, so consumption rises toward the efficient level.
- Rules protecting the vulnerable — e.g. a minimum legal age for smoking — directly restrict harmful over-consumption.

Advantages	Limitations
Direct and enforceable; can guarantee minimum standards. Addresses asymmetric information at source by mandating disclosure.	High monitoring and enforcement costs. Disputes between buyers and sellers over compliance. Effectiveness limited if penalties are weak or evasion is easy.

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3.7 Direct Provision (for public goods and merit goods)

The government funds provision from taxation and supplies the good directly, often free at the point of use. For a public good this is the only way to overcome non-excludability; for a merit good it guarantees consumption up to Qs.

Chain 3.5 — Direct provision of a public good (reproduce this):
Government funds provision via taxation → non-excludability no longer blocks supply → output set at Qs
→ MSB = MSC → allocative efficiency restored

Advantages	Limitations
Guaranteed provision where the private market would supply nothing. Universal access, supporting equity as well as efficiency.	Imperfect information — the government may misjudge Qs and over-provision wastes scarce resources. State producers may be productively inefficient (X-inefficiency) and dynamically inefficient without a profit motive or competition. Strain on the government budget — insufficient tax revenue limits provision, and funding carries an opportunity cost.

3.8 Screening and Signalling (for asymmetric information)

Signalling — by the better-informed party	Screening — by the less-informed party
The informed party reveals credible information so the other can act on it; the receiver interprets the signal and adjusts behaviour. Examples: warranties, seller reputation. Limitation: works only if the signal is credible; a low-value good may not justify the buyer acting on the warranty, so the signal loses force.	The under-informed party acts to uncover hidden characteristics or to induce disclosure, filtering out bad types. Example: insurers requiring a medical check-up. Limitation: screening incurs transaction costs (time, travel); if these are high, buyers may withdraw.

Evaluation: Government Failure and the Efficiency–Equity Trade-off

Government failure: occurs when government intervention produces an outcome worse than the market failure it was meant to correct, so that net social welfare falls. It is the standard evaluative counter-argument to any proposed policy.

4.1 Government Failure — Measurement Error

The efficient tax or subsidy equals the true MEC or MEB, but these external values are intangible and unobservable. Any mis-measurement moves output away from Qs and can leave society worse off than before.

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Chain 4.1 — Government failure from measurement error (reproduce this):
True MEC / MEB unobservable → tax / subsidy ≠ MEC / MEB → equilibrium Q ≠ Qs → residual or new deadweight loss → welfare loss from intervention may exceed the original failure → government failure

4.2 Government Failure — Unintended Consequences

Intervention can create perverse incentives that generate fresh distortions. A price floor is the classic case: set above equilibrium it produces a persistent surplus, and price controls can spawn black markets that misallocate resources.

Chain 4.2 — Government failure from unintended consequences (reproduce this):
Intervention creates perverse incentives → e.g. price floor → surplus or black markets emerge
→ resources misallocated → welfare loss may exceed the original failure → government failure

4.3 The Efficiency–Equity Trade-off

Correcting a market failure can conflict with equity, and the two objectives cannot always be met together. A policy that restores efficiency may impose a disproportionate burden on lower-income groups, so the government must weigh net social benefit rather than pursue efficiency alone.

Chain 4.3 — Efficiency–equity trade-off (reproduce this):
Policy corrects the market failure (e.g. road pricing restores Qs) → but imposes a proportionally heavier burden on lower-income groups
→ equity objective compromised → government must weigh net social benefit → no single optimal solution; the trade-off must be acknowledged explicitly

Evaluation principle for essays: no policy is unconditionally best. Judge each against: (i) how accurately it can be calibrated given information constraints, (ii) its effectiveness given elasticities and enforceability, (iii) its budgetary and opportunity costs, and (iv) its distributional impact on the economic agents – consumers and firms.

The strongest answers weigh a policy mix against these criteria rather than defending a single instrument.

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JC 1 H2 Economics Intensive Revision Workshop

Scoring in Economics requires far more than rote memorisation. Designed for JC 1 students preparing for promotional examinations, this intensive workshop equips students with a structured methodology—Analyse, Select, and Sequence to systematically approach high-weightage Case Study and Essay questions.

Topics Covered

- Price Mechanism
- Firms and Decisions
- Government Microeconomic Objectives and Policies

What to expect

- Learn how to think like an examiner and plan out an answer that answers the question.
- Precise analysis of essay questions that enables you to address the specific points that score marks.
- Detailed case studies with real-life examples.

Helmed by

Ms Summer Wang

Deputy Head of Economics Department

15+ years teaching H2 Economics at a renowned Singapore JC, and deeply familiar with the revised syllabus

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JC 2 A-Level H2 Economics Intensive Revision

Don't let content gaps and key answering skills hold you back for your A-Level exams! This September holiday, gear up for H2 Economics with these essential topics and techniques needed to score in the A-Level paper.

What to expect

- All the essential content, questions, and case studies you need to know to score in the A-Level paper.
- Exam questions to help you apply your knowledge accurately and score marks.
- Comprehensive notes and textbooks to help you consolidate your revision.

Helmed by

Ms Summer Wang

Deputy Head of Economics Department

15+ years teaching H2 Economics at a renowned Singapore JC, and deeply familiar with the revised syllabus

Topics Covered

(Choose 1 run to attend per topic)

Market Structure	**Run 1** 8 Aug (Sat) 4.30pm - 6.30pm Balmoral Plaza, #B1-09	**Run 2** 10 Aug (Mon) 9.30am - 11.30am Balmoral Plaza, #B1-09
Case Studies with Micro- and Macroeconomics Mixed Questions	**Run 1** 27 Aug (Thu) 3pm - 6pm Balmoral Plaza, #B1-09	**Run 2** 29 Aug (Sat) 4.30pm - 7.30pm Balmoral Plaza, #B1-09
Essay Question Selection	**Run 1** 5 Sep (Sat) 4.30pm - 7.30pm Balmoral Plaza, #B1-09	**Run 2** 8 Sep (Tue) 10am - 1pm Balmoral Plaza, #B1-09

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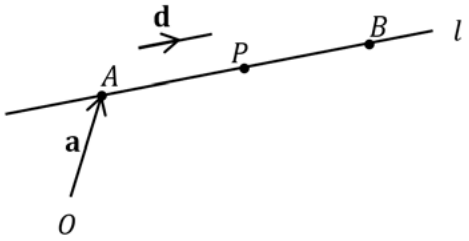
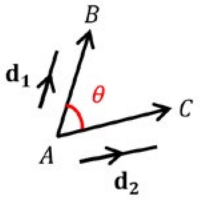
Lines

Equation of a Line

Given $A(2,3,1)$ and $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$,



Vector (Parametric):	$l : \mathbf{r} = \mathbf{a} + \lambda\mathbf{d}, \quad \lambda \in \mathbb{R}$ $= \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$	Requires: 1) Given two points A and B, or 2) Given one point A and one direction vector $\overrightarrow{AB}$.
Cartesian: **(in terms of x, y and z)**	For $l : \mathbf{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} h \\ k \\ 0 \end{pmatrix}$ Make λ the subject, the cartesian equation is $\frac{x-a}{h} = \frac{y-b}{k}, \quad z = c$	
Angle between lines	$\cos\theta = \frac{\mathbf{d_1} \cdot \mathbf{d_2}}{\lvert\mathbf{d_1}\rvert\lvert\mathbf{d_2}\rvert}$ For acute angle, $\cos\theta = \frac{\lvert\mathbf{d_1} \cdot \mathbf{d_2}\rvert}{\lvert\mathbf{d_1}\rvert\lvert\mathbf{d_2}\rvert}$	
Intersection of Lines	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}, \quad l_2 : \mathbf{r} = \begin{pmatrix} 4 \\ -6 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **Step 1:** Combine into single column vector and equate $l_1 = l_2$. $\begin{pmatrix} 2+4\lambda \\ 1-2\lambda \\ 3-\lambda \end{pmatrix} = \begin{pmatrix} 4+2\mu \\ -6+\mu \\ 4-\mu \end{pmatrix}$ **Step 2:** Rearrange and use GC to solve λ and μ. $4\lambda - 2\mu = -2$ $-2\lambda - \mu = -7$ $-\lambda + \mu = 1$ From GC, $\lambda = 2, \mu = 3$ **Step 3:** Sub either λ or μ into l_1 or l_2 to get the intersection point. Sub $\lambda = 2$ into l_1 or Sub $\mu = 3$ into l_2, $\mathbf{r} = \begin{pmatrix} 10 \\ -3 \\ 1 \end{pmatrix}$ ∴ The point of intersection is $(10, -3, 1)$.	

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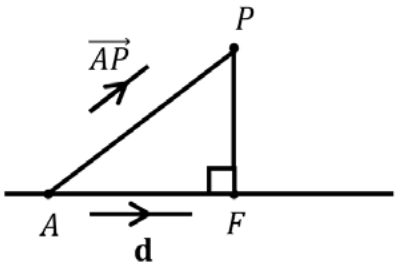
Behaviour of Lines	
For two lines, $l_1 : \mathbf{r} = \mathbf{a} + \lambda\mathbf{d_1}, \lambda \in \mathbb{R}$ $l_2 : \mathbf{r} = \mathbf{b} + \mu\mathbf{d_2}, \mu \in \mathbb{R}$	
Step 1: To check parallel,	
Parallel: $\mathbf{d_1} = k\mathbf{d_2}$ for some $k \in \mathbb{R}$	Non-parallel: $\mathbf{d_1} \neq k\mathbf{d_2}$ for all $k \in \mathbb{R}$
Step 2: If not parallel, check for intersection	
Equate $l_1 = l_2$ to solve for λ and μ.	
If no unique solution, we have **skew lines.**	If there is unique solution, we have **intersecting lines.** Sub λ or μ into l_1 or l_2 to find the point of intersection.

From a Point to Line	
 	
To find the foot of perpendicular, F	
<u>Method 1: Algebraic</u> **Step 1:** F is on the line l Let $\overrightarrow{OF} = \mathbf{a} + \lambda\mathbf{d}$ for some $\lambda \in \mathbb{R}$ **Step 2:** Find $\overrightarrow{PF}$ in terms of λ, $\overrightarrow{PF} = \overrightarrow{OF} - \overrightarrow{OP}$ **Step 3:** Since $\overrightarrow{PF}$ is $\perp$ to l, $\overrightarrow{PF} \cdot \mathbf{d} = 0$ $\Rightarrow$ Solve for λ **Step 4:** Substitute λ into Step 1 to find $\overrightarrow{OF}$	**<u>Method 2: Projection</u>** **Step 1:** Project $\overrightarrow{AP}$ onto line l for vector $\overrightarrow{AF}$ **Step 2:** Use vector addition, $\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF}$
To find perpendicular/shortest distance	
<u>Method 1: If $\overrightarrow{PF}$ or $\overrightarrow{OF}$ is already found</u> Distance $= \left\lvert\overrightarrow{PF}\right\rvert$	**<u>Method 2: Projection $\overrightarrow{AP}$ onto $\overrightarrow{PF}$</u>** Distance $= \left\lvert\overrightarrow{AP} \times \mathbf{d}\right\rvert$

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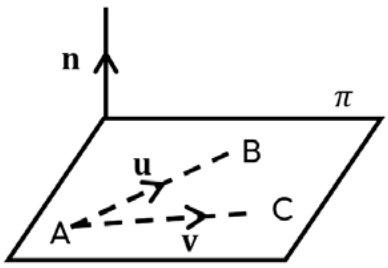
Planes

<table>
<tr><th colspan="3">Equation of Plane</th></tr>
<tr><td colspan="2">Given $A(2,3,1)$, $\overrightarrow{AB}=\begin{pmatrix}1\\-3\\2\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}3\\-2\\1\end{pmatrix}$.</td><td> 
</td></tr>
<tr><td>Vector (Parametric) form</td><td>$\pi : \mathbf{r}=\mathbf{a}+\lambda\mathbf{u}+\mu\mathbf{v}, \qquad \lambda,\mu\in\mathbb{R}$
$\pi : \mathbf{r}=\begin{pmatrix}2\\3\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-3\\2\end{pmatrix}+\mu\begin{pmatrix}3\\-2\\1\end{pmatrix}$</td><td>Condition you need:
1) Given three points, or
2) One point + two parallel vectors.</td></tr>
<tr><td>Scalar form</td><td colspan="2">Step 1: Find normal vector, $\mathbf{n}=\mathbf{p}\times\mathbf{q}$
Step 2: Apply formula:
$\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$
where $\mathbf{a}$ is the position vector of the given point on the plane.
Step 3: Equation of plane,
$\pi : \mathbf{r}\cdot\mathbf{n}=D$</td></tr>
<tr><td>Cartesian form</td><td colspan="2">$ax+by+cz=D$ for some constant D</td></tr>
<tr><th colspan="3">Conversion</th></tr>
<tr><td>Vector form</td><td colspan="2">$\pi : \mathbf{r}=\begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}-1\\0\\5\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\3\end{pmatrix}, \qquad \lambda,\mu\in\mathbb{R}$</td></tr>
<tr><td>⇓</td><td colspan="2">$\mathbf{n}=\begin{pmatrix}-1\\0\\5\end{pmatrix}\times\begin{pmatrix}2\\-1\\3\end{pmatrix}=\begin{pmatrix}5\\13\\1\end{pmatrix}$
$D=\mathbf{a}\cdot\mathbf{n}=\begin{pmatrix}0\\1\\2\end{pmatrix}\cdot\begin{pmatrix}5\\13\\1\end{pmatrix}=15$</td></tr>
<tr><td>Scalar form</td><td colspan="2">$\pi : \mathbf{r}\cdot\begin{pmatrix}5\\13\\1\end{pmatrix}=15$</td></tr>
<tr><td>⇓</td><td colspan="2">Let $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$, $\qquad \begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}5\\13\\1\end{pmatrix}=15$</td></tr>
<tr><td>Cartesian form</td><td colspan="2">$\pi : 5x+13y+z=15$</td></tr>
</table>

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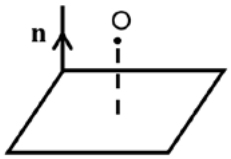
Behaviour of Plane with Plane

Let $\pi_1 : \mathbf{r} \cdot \mathbf{n} = D$ and $\pi_2 : \mathbf{r} \cdot \mathbf{n_2} = D_2$ where $\mathbf{n} = k\mathbf{n_2}$

Perpendicular/Shortest Distance

From origin to Plane

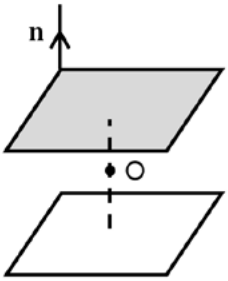


Using length of projection,

Distance from origin

$$= |\mathbf{a} \cdot \hat{\mathbf{b}}| = \frac{|\mathbf{a} \cdot \lambda\mathbf{n}|}{|\lambda\mathbf{n}|} = \frac{|\lambda||\mathbf{a} \cdot \mathbf{n}|}{|\lambda||\mathbf{n}|} = \frac{|D|}{|\mathbf{n}|}$$

Two Parallel Planes



Convert to the same normal first,

$\pi_2 : \mathbf{r} \cdot \mathbf{n_2} = D_2 \Rightarrow \mathbf{r} \cdot k\mathbf{n_2} = kD_2$

$\Rightarrow \pi_2 : r \cdot \mathbf{n} = D_3$

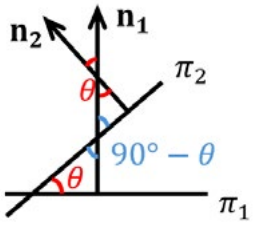
Distance between 2 parallel planes

$$= \frac{|D_3 - D|}{|\mathbf{n}|}$$

Non-parallel Planes

For $\pi_1 : \mathbf{r} \cdot \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} = -4$ and $\pi_2 : \mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = -8$

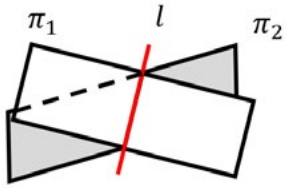
Angle Between Planes



For acute θ,

$$\cos\theta = \frac{|\mathbf{n_1} \cdot \mathbf{n_2}|}{|\mathbf{n_1}||\mathbf{n_2}|} = |\widehat{\mathbf{n_1}} \cdot \widehat{\mathbf{n_2}}|$$

Line of Intersection Between Planes



Step 1: Convert to Cartesian

$$\pi_1 : 2x - 2y + 5z = -4$$
$$\pi_2 : x + 2y - 2z = -8$$

Step 2: Use GC to solve

$$x_1 = -4 - x_3$$
$$x_2 = -2 + \frac{3}{2}x_3$$
$$x_3 = x_3$$

Step 3: Let x_3 be k.

$$\mathbf{r} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + k\begin{pmatrix} -1 \\ \frac{3}{2} \\ 1 \end{pmatrix}, k \in \mathbb{R}$$

$$\therefore l : \mathbf{r} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}, \quad \lambda = \frac{1}{2}k \in \mathbb{R}$$

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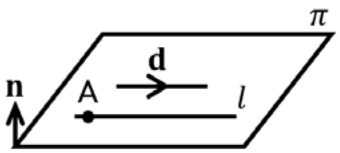
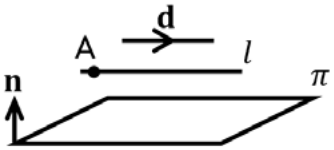
Behaviour of Line and Plane

Let $l : \mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$ where $\overrightarrow{OA} = \mathbf{a}$ and $\pi : \mathbf{r} \cdot \mathbf{n} = D$

Line parallel to Plane

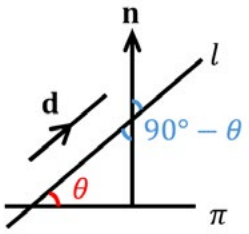
Coincident	Non-coincident
	
Since A is on π ($\mathbf{a}$ satisfies equation of π), $\mathbf{a} \cdot \mathbf{n} = D$ Since l is perpendicular to $\mathbf{n}$, $\mathbf{d} \cdot \mathbf{n} = 0$	Since A is not on π, $\mathbf{a} \cdot \mathbf{n} \neq D$ Since l is perpendicular to $\mathbf{n}$, $\mathbf{d} \cdot \mathbf{n} = 0$

Line Not parallel to Plane

Angle between Line and Plane

For acute angle, since $\cos(90° - \theta) = \sin\theta$,

$$\sin\theta = \frac{|\mathbf{d} \cdot \mathbf{n}|}{|\mathbf{d}||\mathbf{n}|} = |\hat{\mathbf{d}} \cdot \hat{\mathbf{n}}|$$



Intersection between Line and Plane

Given $l : \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $\pi : \mathbf{r} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = 1$

Step 1: Rewrite the equation of line as single column vector

$$\mathbf{r} = \begin{pmatrix} 3 + \lambda \\ -2 - \lambda \\ 7 + 2\lambda \end{pmatrix}$$

Step 2: Substitute into equation of plane and solve λ.

$$\begin{pmatrix} 3 + \lambda \\ -2 - \lambda \\ 7 + 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = 1$$

$$\lambda = -3$$

Step 3: Substitute $\lambda = -3$ into the line,

$$\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore$ The point of intersection is $(0, 1, 1)$.

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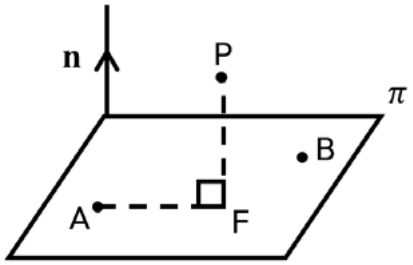
Point to Plane

For point $P(2,-1,0)$ and plane $\pi : \mathbf{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 3$

Foot of perpendicular



Method 1: Algebraic

Step 1: Find equation of l_{PF}

$$l_{PF} : \mathbf{r} = \overrightarrow{OP} + \lambda \mathbf{n}$$

$$l_{PF} : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \qquad \lambda \in \mathbb{R}$$

Step 2: Substitute l_{PF} into π, solve for λ.

$$\begin{pmatrix} 2-\lambda \\ -1+\lambda \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 3$$

$$\Rightarrow \quad \lambda = 2$$

Step 3: Substitute λ into l_{PF}

$$\overrightarrow{OF} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\therefore F(0,1,2)$$

Method 2: Projection

Step 1: Guess a point on the plane

$$\overrightarrow{OB} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

Step 2: Project $\overrightarrow{PB}$ onto $\overrightarrow{PF}$

$$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

$$\overrightarrow{PF} = \left(\overrightarrow{PB} \cdot \hat{\mathbf{n}}\right)\hat{\mathbf{n}}$$

$$= \left[\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \cdot \frac{1}{\sqrt{3}}\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right]\frac{1}{\sqrt{3}}\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$$

Step 3: Find $\overrightarrow{OF}$

$$\overrightarrow{OF} = \overrightarrow{OP} + \overrightarrow{PF} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\therefore F(0,1,2)$$

Perpendicular distance from point to plane

Method 1: If point F is already found

$$\left|\overrightarrow{AF}\right| = \left|\overrightarrow{OF} - \overrightarrow{OA}\right|$$

Method 2: Projection $\overrightarrow{AB}$ onto $\overrightarrow{AF}$

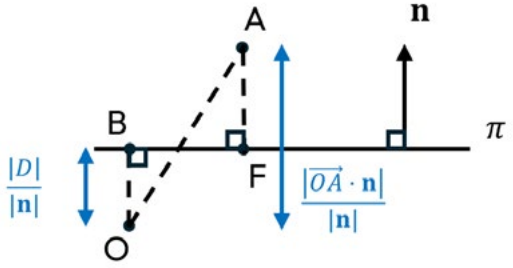
$$\left|\overrightarrow{AF}\right| = \left|\overrightarrow{AB} \cdot \hat{\mathbf{n}}\right|$$

Method 3: Formula

Distance

$$= \frac{\left|\overrightarrow{OA} \cdot \mathbf{n} - D\right|}{|\mathbf{n}|}$$

(similar to finding distance between 2 planes)



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Exam Grade Questions (Vectors)

Practice 1

Relative to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. It is given that λ and μ are non-zero numbers such that $\lambda\mathbf{a} + \mu\mathbf{b} - \mathbf{c} = \mathbf{0}$ and $\lambda + \mu = 1$.

(a) Show that the points A, B and C are collinear. [3]

The angle between $\mathbf{a}$ and $\mathbf{b}$ is known to be obtuse and that $|\mathbf{a}| = 2$.

(b) If k denotes the area of triangle OAB, show that $(\mathbf{a} \cdot \mathbf{b})^2 = 4(|\mathbf{b}|^2 - k^2)$. [3]

D is a point on the line segment AB with position vector $\mathbf{d}$.

(c) It is given that area of triangle OAB is 6 units^2, $|\mathbf{b}| = 10$ and that AOD is $90°$. By finding the value of $\mathbf{a} \cdot \mathbf{b}$, find $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

Practice 2

(a) If $|\mathbf{a}| = 2$, $|\mathbf{a} - \mathbf{b}| = \sqrt{3}$ and $\mathbf{a} \cdot \mathbf{b} = \frac{5}{2}$, use the cosine rule for triangles to find $|\mathbf{b}|$. [3]

(b) It is given that $\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ where a, b and c are constants.

Give the geometrical interpretations of

(i) $|\mathbf{r} \cdot \mathbf{k}|$, [1]

(ii) $|\mathbf{r} \times \mathbf{k}|$, [1]

(iii) Given that $\mathbf{k} \times \mathbf{r} = \mathbf{p}$ and $\mathbf{r} \times \mathbf{p} = \mathbf{k}$, in either order, show that $a^2 + b^2 = 1$ and find the value of c. [4]

Practice 3

A home interior designer is looking to design a pull-down wardrobe lift. To test his concept, the designer built a small-scale model of it, in which points (x, y, z) are defined relative to a corner of the model at $O(0, 0, 0)$. The rods used in the model can be modelled by equations of straight lines.

The main rod can be modelled by the line l_1 with equation $\mathbf{r} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$. Another rod, line l_2, passes through the points $A(5, -5, 8)$ and $B(8, -4, 3)$.

(a) A supporting wire connects B to a point F on l_1 such that $\overrightarrow{BF}$ is perpendicular to l_1. Find the position vector of F. [3]

(b) A retractable rod, line l_R, is placed such that it can be modelled by reflecting l_2 about l_1. Find a vector equation of l_R. [3]

The panels of a small-scale model can be modelled by equations of planes. The main panel, plane p_1, contains the main rod l_1 and an attachment point $C(0, 0, 6)$. A side panel, plane p_2, can be represented by the equation $p_2 : -17x - 37x + 4z = 24$.

(c) Show that p_1 can be represented by the cartesian equation $-x + y + 5z = 30$. [3]

(d) A structural rod, line l_S, needs to be placed at the intersection between p_1 and p_2. Find a vector equation of l_S. [2]

(e) A decorative plank, plane p_3, has equation $p_3 : -3x + \alpha y + 15z = \beta$, where $\alpha, \beta \in \mathbb{R}$. If the distance between p_1 and p_3 is exactly $3\sqrt{3}$ units, find the possible values of α and β. [3]

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You can follow worked examples, but freeze when faced with unfamiliar exam questions. Vectors is one of the highest-weightage A-Level Math topics, yet it's also one of the topics students are least prepared for.

Helmed by
Ms Dilys Han
Dean of Mathematics Department
MOE Teaching Scholar
Former MOE School Subject Head
BSC (Hons) NTU | MA. Columbia University

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- Learn how to approach Vectors questions strategically, recognise key terms, and apply the correct methods, giving you the edge needed to tackle any problem confidently.
- Master this key topic now to get a massive edge before the Promos begin.
- Exam-standard questions that reveal the gap between textbook answers and real exam performance.

Schedule (Choose 1 run to attend each day)

Day 1	**Run 1**	4 Sep (Fri), 10am - 12pm Balmoral Plaza, #B1-09	**Run 2**	11 Sep (Fri), 10am - 12pm Parkway Parade, #06-03
Day 2	**Run 1**	4 Sep (Fri), 1pm - 3pm Balmoral Plaza, #B1-09	**Run 2**	11 Sep (Fri), 1pm - 3pm Parkway Parade, #06-03

Topical Revision Lessons Also Available For

Application of Differentiation	Techniques of Integration	Applications of Integration
Graph	Function	Summation and Recurrence Relations

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JC 2 A-Level H2 Math Intensive Revision

You've revised and practised, but are you still sticking with knowing when or how to apply what you've learnt? Close your gaps in these crucial Math topics this September holidays before the A-Level paper.

What to expect

- Rational functions, conics, and graph transformations drilled to exam standard.
- Know when to use substitution, integration by parts and trigonometric identities without guessing.
- Hybrid questions on domains, inverses and composites that combine topics to prepare you for common exam traps.
- Intensive exam-grade practice that will ensure your methods will become automatic.

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Topics Covered

(Choose 1 run to attend per topic)

Graph	Run 1	4 Sep (Fri), 10am - 12pm Parkway Parade, #06-03	Run 2	7 Sep (Mon), 10am - 12pm Balmoral Plaza, #B1-09
Functions	Run 1	4 Sep (Fri), 1pm - 3pm Parkway Parade, #06-03	Run 2	7 Sep (Mon), 1pm - 3pm Balmoral Plaza, #B1-09
Applications of Integration	Run 1	7 Sep (Mon), 1pm - 3pm Parkway Parade, #06-03	Run 2	11 Sep (Fri), 1pm - 3pm Balmoral Plaza, #B1-09
Summation and Recurrence Relations	Run 1	8 Sep (Tue), 4pm - 6pm Balmoral Plaza, #B1-09	Run 2	10 Sep (Thu), 10am - 12pm Parkway Parade, #06-03

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Overview of Application Question (AQ)

Sample AQ Question Formats

2024 A Levels: The reading passages cover a range of views about working from home. How far do you agree that working from home offers more advantages than disadvantages?

Support your answer with reference to:
- the ideas and opinions from at least one of the reading passages
- examples drawn from your experiences and that of your society.

2025 A Levels: The reading passages cover a range of views about AI. How far do you agree that humans should be concerned about the rise of AI?

Support your answer with reference to:
- the ideas and opinions from at least one of the reading passages
- examples drawn from your experiences and that of your society.

Structures

An AQ is typically written in 30 min, and has a **macrostructure** similar to a short essay:

Components	Comments
Introduction	1-2 sentences to state overall stand
Body Para 1	Typically from author 1
Body Para 2	Typically from author 2 / 3
Conclusion	Overall stand, possibly with weighing

The **microstructure** of each body paragraph is also similar to that of a PEEL essay paragraph:

Components	Comments
Reference	Quote directly from the passage
Explanation	Unpack the quote's point in your own words
ATQ	Agree or disagree with the point
Evaluation	Explain why you agree / disagree, with examples

Skills

Extracting Points

- When it comes to extracting points, there are three considerations for what to extract to later evaluate throughout the paragraph:

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- A perfect point is what the question defines as a point (i.e., your point must be relevant to the question being set).
- A perfect point is what the author intends to be a point (i.e., ideally your point should align with the author's overall stand).
- A perfect point is a point, not a general stand or a specific example.

Evaluating Points

- On what basis can we determine whether a point is true or untrue of our society, or defend why we agree or disagree with the point?
  - A characteristic of our society provides an **explanation** for why the point is true or untrue *of* Singapore.
  - A case study or statistic from our society provides **evidence** that the point is true or untrue *in* Singapore. We can choose anything from public news stories to personal anecdotes (try not defaulting to personal anecdotes though, given that concrete examples are preferred) to illustrate our agreement or disagreement.

Strategies

Strategy 1: single context, no weighing

Macro Component	Micro Component	Example
Introduction	Stand	I agree that X is good.
Para 1	Reference	Author 1 says "…"
	Explanation	This means X is good because Y.
	ATQ	I agree.
	Evaluation	Singapore is A society… For example…
Para 2	Reference	Author 2 says "…"
	Explanation	This means X is bad because Z.
	ATQ	I disagree.
	Evaluation	Singapore is B society… For example…
Conclusion	Stand (no weighing)	It is true that X has Y benefit. It is not true that X has Z cost. Therefore, X is good.

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Strategy 2: single context, weighing

Macro Component	Micro Component	Example
Introduction	Stand	I agree that X is good.
Para 1	Reference	Author 1 says "…"
	Explanation	This means X is good because Y.
	ATQ	I agree.
	Evaluation	Singapore's society is A… For example…
Para 2	Reference	Author 2 says "…"
	Explanation	This means X is bad because Z.
	ATQ	I agree.
	Evaluation	Singapore's society is B… For example…
Conclusion	Stand (weighing)	It is true that X has Y benefit. It is also true that X has Z cost. Ultimately, X is good because Y > Z.

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Evaluation Shortcuts

To answer the Application Question well, you must familiarise yourself with the characteristics of Singapore and local examples.

The acronyms CEFGKPPO and AAMME offer some (non-exhaustive) key traits of Singapore society that you can apply in your AQ, if relevant.

What kind of society is Singapore?	**CEFGKPPO** • Competitive • Everchanging • Fast-paced • Globalised • Knowledge-based • Pragmatic • Profit-driven • Open
	AAMME • Asian conservative • Ageing • Meritocratic • Multiracial / Multicultural • Educated

If your conclusion requires weighing the two body paragraphs (See Strategy 2), you will then need to determine which body paragraph is more representative of our lived experiences as Singaporeans. You can do this by picking from any one of the criteria below:

How do we weigh the two views?	• **Exception** vs **Norm** / **Majority** vs **Minority** • **Future** vs **Past** vs **Present** / **Long-term** vs **Short-term** • **Ideal** vs **Real** / **Theory** vs **Practice**

So for instance, if the first body paragraph is the majority while the second body paragraph is the minority, then you will, on balance, side with the first body paragraph as it represents our lived experiences more.

Of course, this weighing will **not** be necessary if you go with Strategy 1.

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AQ Practice 1

Application Question
The reading passages cover a range of views about migrants. How far do you agree that we should embrace migrants? Support your answer with reference to: • the ideas and opinions from at least one of the reading passages • examples drawn from your experiences and that of your society.

While there will be multiple full passages provided in the exam, AQ Practice 1 will only focus on a short excerpt from one of the passages that talks about the benefits that migrants can bring.

Read the excerpt below, and attempt the self-practice section on the next page.

Excerpt
So why not make a virtue of it? After all, we need them. We need them to care for the old and to look after the young, to allow mothers back to work and free up time in our busy lives. We need them even more because every foreigner we recruit adds to the demand for people willing to do menial work. Indeed, because low-skilled workers are in relatively short supply in rich countries, while highly skilled workers in general are not, we potentially have most to gain from letting them in. Perhaps most importantly, in a world where different perspectives are at a premium, immigrants stimulate innovation and economic growth. Just look at flourishing cosmopolitan cities like London and New York. They are the future. Allowing people to come and work in rich countries legally would have other big benefits too. It would save lives: more migrants have died trying to cross the US-Mexico border since 1995 than were killed in the 9/11 attacks.

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Self-Practice	**Identify a point from the excerpt above.** **Fill in the components of your AQ response.**
Reference:	“ ”
Explanation:	This means that…
ATQ:	I agree / disagree with the author [Circle/Underline your stance].
Evaluation	

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Sample Response

Body Paragraph
Reference: The passage states that “every foreigner we recruit adds to the demand for people willing to do menial work", which "are in relatively short supply in rich countries".
Explanation: This means that the author is in favour of welcoming migrants, as they expand the labour force by taking on physically demanding jobs often shunned by locals.
ATQ: I agree with the author.
Evaluation: This perspective holds particular relevance in Singapore, a nation shaped by a **pragmatic** approach to cost and benefit. With a highly **educated** workforce, many Singaporeans naturally gravitate towards specialised, well-paying roles, steering clear of low-wage, low-status jobs that offer little prestige. Undoubtedly, the nation’s **emphasis on self-reliance and meritocratic achievement** has fostered a collective ambition to strive for greater heights. However, this mindset presents challenges for the economy, as sectors reliant on labour in the so-called “4D” jobs — dirty, dangerous, dull, and demeaning — experience acute shortages. In this context, migrants emerge as invaluable contributors. Unencumbered by the rigid expectations of status and remuneration that often shape local career choices, they undertake these vital roles with resilience and determination. When these opportunities provide them with far better prospects than those available in their home countries, the outcome is undeniably mutually beneficial, fostering both economic advancement and social stability. This point is aptly exemplified by Singapore's construction industry, which remains heavily dependent on migrant labour. As of December 2023, the Ministry of Manpower reported 441,100 work permit holders employed across the construction, marine shipyard, and process sectors, highlighting the industry's reliance on foreign workers. In stark contrast, data from Statista indicated that only around 94,300 residents were employed in the construction sector during the same period. This significant disparity underscores the pivotal role of migrant workers in sustaining the industry, as they predominantly occupy labour-intensive and physically demanding roles that are often shunned by the local workforce.

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AQ Practice 2

Passage 1: *An author considers the real cost of sharing personal information online.*

1 "If you are not paying for the product, you are the product." This oft-repeated phrase has been circulating the internet over the past decade, though the idea certainly predates this. This truism raises a worrying question — are we willing participants in the biggest bargain on the internet, or unwitting accomplices in its biggest scam?

2 Each time we routinely use a search engine, casually tweet a message or upload a photograph or video, we serve up our personal information to the host of these services. No company can operate without income; their services are monetised either by marketing portions of web space to advertisers or through peddling our precious data to any organisation with an interest in re-purposing our information.

3 Collectively, we use internet search engines billions of times each day, providing no shortage of personal data which can then be traded. Our unquenchable thirst to share every aspect of our lives on social media translates to ample commercial opportunities for exploitation. Most social media sites do not charge their users any fees, yet are now worth billions of dollars. Many have flourished to the point where over a billion people now regularly use them. The evidence suggests this is a business model that works.

4 Although users may be aware that information is constantly collected about their location, demographics, behaviour and habits**,** they are usually ignorant as to how it will be employed or who has access to it. This lack of transparency is worsened by the fact that the terms of service on many social media sites are not written for the general public's consumption. Many run to tens of thousands of words and would take several hours to read, and many more to comprehend. Adding insult to injury, most sites have such broad licensing terms, allowing the host company free use of our personal material whilst still paying lip service to our 'ownership' of it. Your treasured holiday photo shared on your social media could end up being featured in a hotel chain's global advertising campaign. You would be none the wiser, and receive no remuneration, while the website would pocket the licence fees for using your material. Would users disclose their lives so freely if they knew that their creative output such as blogs, photos and videos could be modified, published, distributed or retained without their consent?

5 Some users assert that the exploitation of personal data and targeted advertising is not a new phenomenon. To cite only one example, since the early 1990s, many

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stores have launched loyalty cards. Such stores are happy to take our money for their goods, whilst constantly monitoring our choices in an attempt to sell us further products. We gladly sign up to these programmes all in return for a tiny discount or voucher to spend, once again, at their store.

6 So why is there heightened concern about our privacy now? The simple reason is that nothing matches the tentacles of the internet for its insidious spread and reach. Even if you delete your account on social media or other websites, once your personal information is out there, there really is no way to rein it back to stop it from being circulated or used.

7 So the next time you are tempted by a 'free' social media application or web service, remember my modern interpretation of the ancient Roman warning to consumers — "if you are not the buyer, then you must beware!"

[End of Passage]

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Passage 2: *An author argues the case for collecting personal data.*

1 A single ant or bee isn't smart, but their colonies are. The study of swarm intelligence is providing insights that can help humans create complex management systems, from the routing of delivery vehicles to the controlling of groups of robots or containing the spread of epidemics.

2 Swarm intelligence is defined as the collective behaviour of decentralised, self-organised systems. The agents in these systems interact randomly and their interactions contribute to the emergence of 'intelligent' communal behaviour. A colony can solve problems that are far beyond the abilities of a single ant. These include problems of survival such as defending the colony or discovering the best route to a food source. There is no decision-maker: each ant adjusts its behaviours based on countless interactions with others in its community.

3 Keen to harness the power of this natural phenomenon, academics have been investigating possible human applications of this model. An experiment conducted by university researchers has demonstrated that networked users can collaborate in real-time to answer questions, rank decisions, generate ideas and even express opinions. Through an online system, participants were set tasks that had verifiable results, ranging from predictions of winners of sports events to controversial political questions. Each participant demonstrated the ability to collectively explore the group's decisions against their own and reach compromises. Just like biological swarms: groups of people outperform their individual members and achieve optimal outcomes.

4 Utilising the power of connections, like those made through bee pheromones or whales singing to one another, is not just something limited to nature. Anyone who has an online presence or a mobile phone connected to the internet is continuously receiving information and broadcasting their personal data**.** This data might be their current physical location or choices of goods and services ordered and consumed. Some of this data is freely offered by the user and the rest gathered as a matter of course in the non-stop stream between service provider and user. Take the scenario of thousands of passengers gathered and stationary in one location due to a train suddenly breaking down: instantly and automatically, many passengers will use their phones to communicate to others, whilst their phones will send data signals tracking their location. Messages posted by these stranded people to social media sites might highlight the incident, providing real-time updates that may be checked and would warn others about the incident, providing real-time updates that may be checked and would warn others of the incident, providing real-time updates that may be checked and would warn others of the incident. Taxi and bus companies pick up on this spike in demand via global positioning systems and dispatch more vehicles to the area. Like ants interacting

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with one another, both the independent actions of passengers and the constant streaming of their personal data produces swarm intelligence that is used to benefit everyone, and 'human swarming' is in action.

5 It is not difficult to imagine how the mundane scenario of travel delays could be scaled up to address emergency situations such as the management of a natural disaster or stopping the spread of a deadly disease — from identifying the worst-hit areas to directing assistance and allocating medical resources. During such crises our personal internet devices will still function and we now have the power at our fingertips to 'swarm' in a disaster. The more information that is collected, the more powerful the 'swarm' response can be. We willingly share our personal views and information, and it is only right that it is used for the greater good of society. It is only through collecting all data that 'the whole is greater than the sum of its parts' (Aristotle). So, if we are looking for a role model in a world of chaos and uncertainty, we could certainly do worse than to copy the humble ant.

[End of Passage]

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Application Question
The reading passages cover a range of views about the collection and use of personal data. How far do you agree that the collection and use of personal data bring more benefits than harm? Support your answer with reference to: • the ideas and opinions from at least one of the reading passages • examples drawn from your experiences and that of your society.

Self-Practice	**Identify a point from either Passage 1.** **Fill in the components of your AQ response.**
Reference:	“ ”
Explanation:	This means that…
ATQ:	I agree / disagree with the author [Circle/Underline your stance].
Evaluation	

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Self-Practice	**Identify a point from either Passage 2.** **Fill in the components of your AQ response.**
Reference:	“ ”
Explanation:	This means that…
ATQ:	I agree / disagree with the author [Circle/Underline your stance].
Evaluation	

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Sample Response

Introduction
I **disagree** in Singapore's context that the collection and use of personal data bring more benefits than harm.

Body Paragraph 1
Reference: Passage 1 states that people are "usually ignorant as to how [their data] will be employed or who has access to it".
Explanation: This means that leaving breadcrumbs of personal information online can pose risks to individuals as it can be used in ways that they are unaware of and have no control over.
ATQ: I agree with the author.
Evaluation: Singaporeans **prioritise convenience and speed** because of their **fast-paced** lifestyles. This means that many Singaporeans keep their online accounts permanently logged into, facilitating their ease of navigating in and out of different websites without really thinking twice about the information they are giving away. Unfortunately, in March 2025, Home Team Singapore was hit by a ransom ware attack where the data of employees and former employees had been accessed from data. The concern was that the data breach would leave users vulnerable to phishing and unauthorised transactions. It is often only when such breaches are identified that users are aware of the risks of their information existent online. This reflects the risks that we unknowingly put ourselves in, all for the sake of convenience and ease of access to information online.

Body Paragraph 2
Reference: Passage 2 claims that "a colony can solve problems that are far beyond the abilities of a single ant".
Explanation: This means that the collection and use of personal data can be helpful because it enables people to solve problems collectively, rather than relying solely on an individual's ability or resources.
ATQ: I agree with the author.

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Evaluation: As Singaporeans are generally **digital natives**, this means that we are adept at navigating online spaces like the Internet. As a collective and democratised platform for the exchange of information, the Internet enables Singaporeans to crowdsource ideas, advice and solutions from a diverse community of users. Young people in Singapore have readily embraced this collaborative environment. Their familiarity with digital platforms lets them seek assistance and access resources efficiently. 18-year-olds in Singapore whether taking the International Baccalaureate or the GCE A levels are all keenly cognizant of Reddit's threads, R/IB, R/A Levels where they can ask questions and pose problems and more often than not, the collective hive of the sub reddit is able to provide them the answers or solutions they seek. It is this interaction with the collective that augments the powers and understanding of the individual, far beyond what they themselves can accumulate on their own.

Conclusion (Weighing)

Ultimately, in an **ideal** world, the collection and use of personal data would bring about only benefits to Singapore. However, this assumes that such information is always handled in a responsible way—as assumption that does not reflect **reality**. Until the risks associated with collecting and using personal data can be eliminated, I **disagree** that such a practice brings more benefits than harms.

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CURRENT AFFAIRS	SINGAPORE'S LEGAL SYSTEM

Possible Essay Questions

1. "Education, not censorship, is the solution to misinformation." To what extent do you agree? (ASRJC 2024)
2. "The government should never interfere with people's private lives." To what extent is this true of your society? (CJC 2024)
3. To what extent is the success of a government measured by its ability to maintain law and order? (RVHS 2024)
4. In today's troubled world, should governments impose more restrictions over the media? (SAJC 2024)

Introduction: The Objectives of Legislation

Generally, the fundamental purpose of the government is to ensure and advance the well-being of society at large. Legislation can advance this goal in two ways.

Firstly, the law is used to outline and enforce a framework of "dos" and "don'ts" that the citizenry should abide by to ensure the stability, security and well-being of society at large. The law can be a powerful way of advancing collective, nationwide objectives as the state possesses the ability to exercise compulsion, such as punishing citizens for violating the law and jeopardising the welfare of society.

Secondly, legislation can also take the form of passing bills and policies to authorise the provision of public goods, services and resources to alleviate specific shortages in the population and raise the people's standard of living. This is crucial as, in many cases, only the state possesses the resources to ensure social welfare, like the ability to provide healthcare subsidies.

It is key to note, however, that legislation is not static: it is a constantly evolving process which adapts to shifting mindsets and the emerging challenges a country faces.

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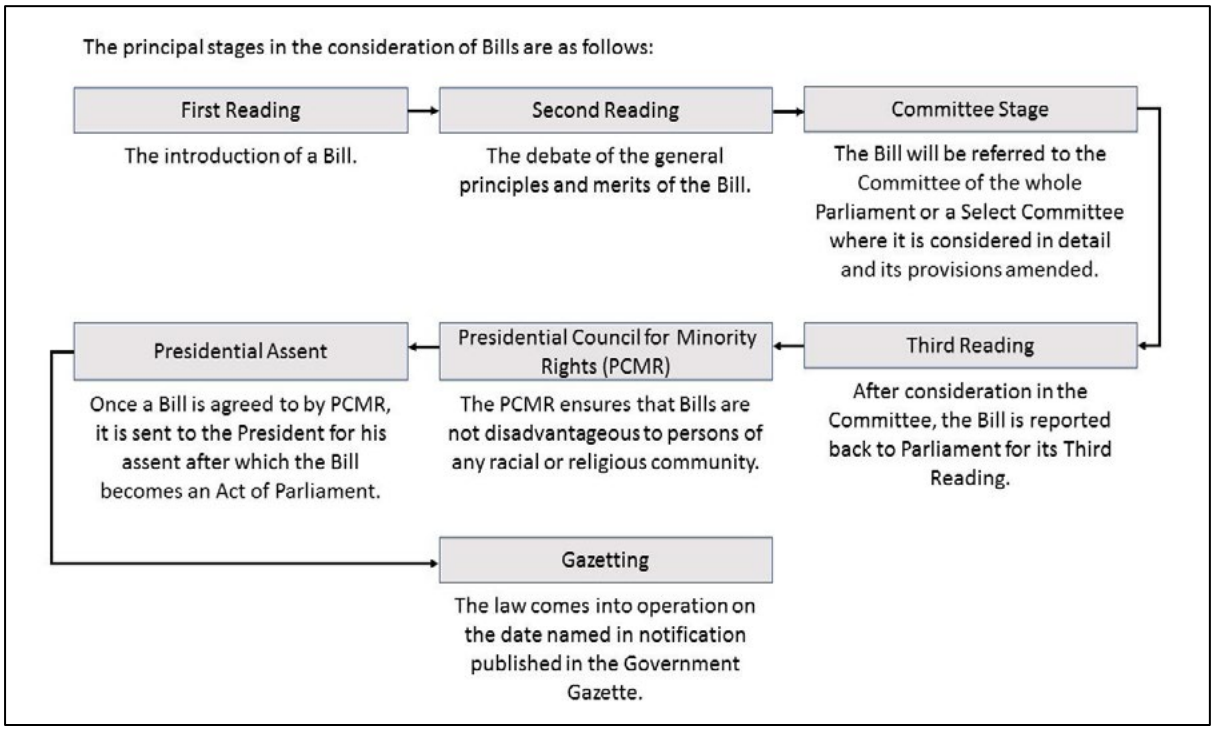
How Does Singapore's Legal System Work?

1. **Singapore's legal system comprises three branches: the Legislature, the Executive and the Judiciary.**

- In Singapore, the Legislature makes the laws of the land, and is the highest law-making authority in the country. It comprises the Parliament and President.
- Notably, all bills must go through three readings in Parliament and receive the President's assent before they become laws.
- Below is a flowchart detailing the steps taken before a law is officially passed.



Source: Parliament of Singapore

- The Executive, on the other hand, enforces the law the legislature has enacted by devising policies and regulations to ensure that the laws are implemented effectively. It comprises the Cabinet and the myriads of public service agencies under their charge.
- Led by the Prime Minister, the Executive is sometimes known as the government.
- Finally, the Judiciary interprets the laws that the Legislature has enacted, should the cases go to court. It includes the court system and all officers working for the courts.
- Another function of the Judiciary is to check the exercises of legislative and executive power. The courts can review Parliament-passed laws and subsequently decide if they are constitutional. Laws decided to be unconstitutional will be struck down.

➢ This three-branch structure is known as the separation of powers. It is designed to ensure that there are checks and balances to prevent abuse of power. This arrangement is set in the Constitution, which the Parliament of Singapore calls the "supreme law of the land."

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Notable Laws

1. **The Protection from Online Falsehoods and Manipulations Act (POFMA) was passed to improve the quality of information circulating online, as well as guard our nation's security against the threats posed by misinformation.**

- In the light of the influence of fake news on the results of the 2016 elections in the United States, fake news was identified as a threat to democratic institutions in Singapore by the authorities. Against the backdrop of the 2020 General Elections, the Select Committee on Deliberate Online Falsehoods was created, along with the passing of POFMA in 2019, to prevent fake news from insidiously swaying the public's opinion before the polling day.

- The Committee was conceptualised as a specialised governmental body to be entrusted with the task of debunking fake news, curbing its ramifications such as the negative influences it can exercise over public opinion and the disruption of public order.
- Singapore is highly vulnerable to damage caused by malicious foreign actors. Online platforms, in particular, are an effective tool for these groups or individuals to perpetuate their malignant agendas due to the high rate of digital penetration in Singapore.
- One way these actors could potentially jeopardise our national security is via publishing misinformation to sow discord among our multicultural population.
- Singapore hence requires adequate legislation to keep the spread of such malicious and manipulative content under control.
- Asides from curbing the spread of misinformation on the Internet, POFMA also aims to improve the quality of the social media environment by strengthening the framework for identifying and taking legal actions against cybercrimes like doxxing.[2]

➢ However, the efficiency of POFMA is debatable as misinformation can spread rapidly on the Internet, and can be difficult to discern from other news due to the overwhelming volume of content on the Internet.

➢ In addition, many have voiced concerns about how POFMA could arm the government with even more ammunition to erode the people's rights to freedom of expression. These concerns are rooted in observations that the incumbent party in Singapore has a history of utilising the legislature to censure various forms of political opposition.

➢ For example, before the rise of independent journalism in Singapore enabled by the Internet, the Printing Presses Act placed all forms of the press under government control, as all news agencies could only operate with the approval and license of the government.

➢ The Sedition Act, beyond the justifiable intention of suppressing insensitive racial and religious remarks, was an insidious means of curtailing the freedom of Singaporeans to comment on the government and its policies. Even till today, the Sedition Act continues in

[2] Doxxing refers to the practice of searching for and publishing the private information of an individual online, typically with malicious intent.

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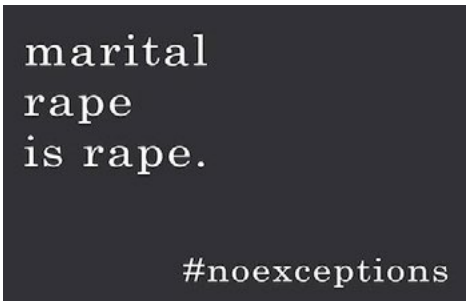
place of the Printing Presses Act, to curtail the issues that independent news agencies in Singapore can publicly discuss.

- Similarly, while POFMA ostensibly aims to tackle digital misinformation, some worry that comments by online users on subjective issues (such as a national policy) which are inimical to the government's interests could be taken down by being labelled as "misinformation".
- An ambiguous definition of misinformation, alongside a rather opaque process of how misinformation is identified, could make POFMA another insidious method of clamping down on political opposition and freedom of expression.

2. Marital rape was made a legal offence, a significant move towards protecting women against sexual abuse more effectively.



- Singapore's legal system also permits law amendments, a notable one being the Women's Charter, which originally outlined marital and gender equality policies passed in 1961.
- Section 375(e) of the Singapore Penal Code—which previously gave husbands who had non-consensual intercourse with their wives immunity from legal repercussions—was repealed to criminalise sexual misconduct within wedlock.
- Against the backdrop of the global #MeToo movement (where victims of sexual assault stood in solidarity), criminalising marital rape is a progressive policy which ensures that women's rights would not be undermined, even during marriage.
- However, it is key to note that the law is not a panacea for all wrongs in our society, and gender equality in Singapore is very much a work in progress.
- Advocates for women's rights point out how cases of sexual violence and abuse against women continue to occur frequently in Singapore (despite strict legal penalties for such offences), demonstrating the discriminatory and patriarchal attitudes that persist here.
- Rather than relying solely on the law to provide a "cure" by punishing perpetrators for their wrongdoing, we should focus on preventing such incidents from occurring in the first place, as punishing the perpetrators cannot undo the harm suffered by the victims.
- There needs to be a paradigm shift in our perceptions of gender and an overhaul of our discriminatory attitudes towards women.

- However, a point of hope lies in how the populace is slowly becoming more educated on both the more overt and insidious ways women are victimised by the more extensive system and harmful cultural attitudes.
- Amidst a new wave of advocacy globally, public opinion has shifted, and people are now placing pressure on authorities and the larger system to protect their basic and essential rights, as reflected in the repudiation of section 375 (e).
- In addition, just like how changing mindsets amongst the citizenry can mould the law, the law can mould public opinion as well. With marital rape being acknowledged as a criminal offence, it is hoped that the public will start recognising that the different kinds of abuses towards women that should not be condoned.

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3. The Foreign Interference (Countermeasures) Act (Fica) was implemented to enhance the government's ability to identify, prevent and neutralise attempts by foreign actors to interfere in Singapore's politics.

- Passed in Parliament in late 2021, Fica officially came into effect on December 29, 2023.
- According to a 2024 article by The Straits Times, Fica is a law empowering the authorities to deal with covert efforts by hostile foreign entities to interfere in a nation's domestic political affairs.

- Such forms of interference include "artificially amplifying certain views to manipulate public opinion on hot-button issues, stir up discord, or undermine confidence in institutions."
- The law also aims to combat subversion and influence operations through the use of local proxies (i.e., using local channels or agencies to advance foreign agendas).
- Why was it introduced then? According to a Ministry of Home Affairs (MHA) news release, foreign interference posed a big risk to "(Singapore's) sovereignty and national security," and that there have been "many instances" where "hostile information campaigns" have been carried out against other countries in recent years.
- An example cited by the MHA was a former Australian senator who had, after "receiv(ing) donations from a foreign billionaire," subsequently "advocated for the foreign country's position on a sensitive issue of national security, contradicting his own political party's official position."
- It is key to note that the law does not target Singaporeans or other local entities that voice their opinions, unless they are being utilised by foreign actors as proxies for interference.
- Instead, to protect local politics, the law applies to two specific categories of people and organisations: defined Politically Significant Persons (PSPs) and designated PSPs.
- Defined PSPs include actors that are directly involved in Singapore's political processes. Thus, political parties, political office holders, and members of Parliament (including non-constituency MPs and nominated MPs) come under this category. This includes the ruling party and the opposition parties (as well as all its members).

- Designated PSPs include actors whose activities could be aimed at political ends, and that it is in the public's interest that they be subjected to Fica's measures to mitigate the risk of foreign interference.
- What are these two groups subjected to? Defined PSPs face stricter countermeasures. These include (1) mandatory maintenance of a separate bank account to receive political donations, (2) mandatory declaration of any donation exceeding $10,000 and (3) a ban on accepting donations and volunteer labour/services from foreign entities or individuals.

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- Designated PSPs, on the other hand, are only required to report political donations and disclose foreign affiliations. However, if the risk of foreign interference is deemed large enough, they will be subject to the same countermeasures that apply to defined PSPs.
- In February 2024, Fica was invoked for the first time against naturalised Singaporean (Hongkong-born) businessman Philip Chan Man Ping, who then became a designated PSP.
- According to MHA, Chan "has shown susceptibility to be influenced by foreign actors, and willingness to advance their interest," and that his "activities are directed towards a political end."
- Indeed, according to a 2024 article by The Straits Times, Chan had declared to a Chinese media outlet that, "It is our duty as overseas Chinese to tell China's story well, and to spread and pass on the marvellous traditional Chinese culture while we are abroad."
- He was also taking on leadership positions in politically significant entities: he was part of the Bukit Timah Community Club management committee, and was President of the Kowloon Club, which helps new immigrants integrate into Singapore society.
- Surprisingly, in July 2024, MHA said that it intended to label the National Trades Union Congress (NTUC) as a designated PSP.
- MHA said that it was in the public interest to do so, given that NTUC has a "close nexus and symbiotic relationship" with the PAP.

ntuc
National Trades Union Congress

- In addition, a substantial portion of NTUC's work is inherently political (i.e., political ends), since it focuses on matters like industrial relations and workers' rights and well-being.

➢ Some caution that Fica could have a chilling effect on legitimate entities like academics studying contentious issues and foreigners commenting on Singapore politics.
➢ In reply, MHA clarified that Fica is not meant to apply to foreigners who express views on Singapore's domestic politics local matters in an "open" and "transparent" manner.
➢ The Workers' Party (WP) and other observers have also argued that the broad language of Fica could leave room for potential abuse by current or future governments.
➢ In reply, Minister for Home Affairs K. Shanmugam asserted that the risk of rogue foreign interference significantly outweighs the risk of a rogue government abusing its authority.

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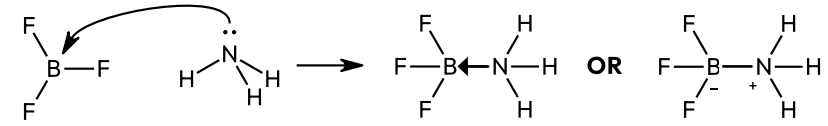
Chapter 12. Acid-Base Equilibrium

Theories of Acids & Bases

NH_3 is **NOT** an Arrhenius base.

	Acid	Base
Arrhenius Theory	a substance that already contains H^+ and when placed into water, dissociates to give $H^+(aq)$. $HCl(g) + aq \rightarrow H^+(aq) + Cl^-(aq)$	a substance that already contains OH^- and when placed into water, dissociates to give $OH^-(aq)$. $KOH(s) + aq \rightarrow K^+(aq) + OH^-(aq)$
Brønsted-Lowry Theory	H^+ (or proton) donor	H^+ (or proton) acceptor
	A **conjugate acid-base pair** differs by a H^+ (e.g. HCl and Cl^-). The stronger the acid (or base), the weaker the conjugate base (or acid). e.g. $HCl(g) + NH_3(g) \rightarrow NH_4^+Cl^-(s)$ HCl is a Bronsted-Lowry acid as it donates a H^+ to give conjugate base Cl^-. NH_3 is a Bronsted-Lowry base as it accepts a H^+ to give conjugate acid NH_4^+.	
Lewis Theory	electron pair acceptor (e.g. electrophiles, metal ion/atom in complexes)	electron pair donor (e.g. nucleophiles, ligands)
	 Lewis acid Lewis base	

Factors Affecting pH

$$pH = -\log_{10}[H_3O^+] \qquad [H_3O^+] = 10^{-pH}$$

$$pOH = -\log_{10}[OH^-] \qquad [OH^-] = 10^{-pOH}$$

- pH and pOH are dependent on many factors – concentration of the acid or base, basicity of the acid or base, and strength of acid or base.
- The **concentration** of acid/base refers to the amount of acid/base that is dissolved in 1 dm^3 of water, and is measured in mol dm^{-3}.
  - A concentrated acid has a higher $[H^+]$ (and lower pH) than the diluted same acid.
  - A concentrated base has a higher $[OH^-]$ (and higher pH) than the diluted same base.

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- The **basicity** of acid/base refers to the maximum number of H^+/OH^- ions dissociated per formula unit of acid/base.

monobasic acid	dibasic acid	monoacidic base	diacidic base
HC*l*	H_2SO_4	NaOH	$Ca(OH)_2$
CH_3COOH	HOOC–COOH	NH_3	N_2H_4

- A dibasic strong acid has a higher $[H^+]$ (and lower pH) than the monobasic strong acid.
- A dibasic strong base has a higher $[OH^-]$ (and higher pH) than monobasic strong base.

Strength of Acids and Bases

strong acid	weak acid
• Dissociates **fully** in solution to give H^+ • Most mineral acids are strong acids. • e.g.: HI, HBr, HC*l*, H_2SO_4, HNO_3 etc. $HBr(g) + H_2O(l) \rightarrow H_3O^+(aq) + Br^-(aq)$	• Dissociate **partially** in solution to give H^+ • Most organic acids are weak acids. • e.g.: CH_3CO_2H, C_6H_5OH, HCN, HF etc. $HCN(aq) + H_2O(l) \rightleftharpoons CN^-(aq) + H_3O^+(aq)$
strong base	**weak base**
• Dissociates **fully** in solution to give OH^- • e.g. NaOH, KOH, $Ba(OH)_2$ etc. $NaOH(s) + aq \rightarrow Na^+(aq) + OH^-(aq)$	• Ionises **partially** in solution to give OH^- • e.g. NH_3, CH_3NH_2, Na_2CO_3 etc. $NH_3(aq) + H_2O(l) \rightleftharpoons NH_4^+(aq) + OH^-(aq)$

Ammonia ionises in water, not dissociates; dissociation is the breaking apart into ions that were already present in it.

Worked Example

Why does 1 mol of strong acid and 1 mol of weak acid require the same amount of NaOH for complete neutralisation?

Consider 1 mol of strong acid H**A** and weak acid H**B** each:

$$H\mathbf{B}(aq) \rightleftharpoons H^+(aq) + \mathbf{B}^-(aq) \text{ --- (1)}$$

- Initially, concentration of H^+ ions in a solution of H**B** is low. Upon reaction with NaOH, the H^+ ions are used up, causing the concentration of H^+ ions to drop.
- By Le Chatelier's principle, the position of equilibrium (1) shifts right to increase the concentration of H^+ ions. These newly dissociated H^+ ions can continue to react with NaOH, causing the concentration of H^+ ions to decrease.

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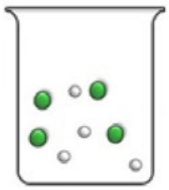
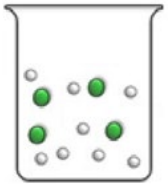
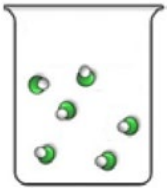
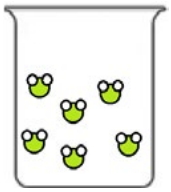
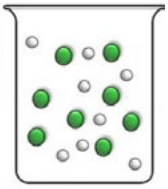
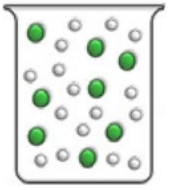
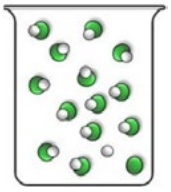
- This process continues until all the H**B** are fully ionised and neutralised, resulting in same amount of NaOH required as H**A** for complete neutralisation.

> ⚠ The amount (moles) of base required for neutralisation depend on the amount of acid present and the stoichiometric ratio; they are independent of acid/base strength. Strength only affects the [H^+], and hence the rate of reaction.

	Strong (full dissociation)		**Weak (partial dissociation)**	
Dilute	monobasic 	dibasic 	monobasic 	dibasic 
Concentrated	monobasic 	dibasic 	monobasic 	dibasic

- Quantitatively, one can look at the extent of dissociation.

$$\text{extent of dissociation, } \alpha = \frac{\text{[acid ionised]}}{\text{[initial acid]}}$$

- If $\alpha = 1$, the acid is strong.
- If $\alpha < 1$, the acid is weak.

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pH Calculations of Different Solutions

System	Acid/Acidic	Base/Basic
① **Strong Acids / Bases**	$pH = -\log_{10}[H^+]$	$pOH = -\log_{10}[OH^-]$
② **Weak Acids / Bases**	**Method 1**: set up ICE table → use K_a to solve for $[H^+]$ at equilibrium $K_a = \frac{[H^+]^2}{[acid]}$ **Method 2**: $pH = -\log_{10}\sqrt{K_a \times [acid]}$	**Method 1**: set up ICE table → use K_b to solve for $[OH^-]$ at equilibrium $K_b = \frac{[OH^-]^2}{[base]}$ **Method 2**: $pOH = -\log_{10}\sqrt{K_b \times [base]}$
③ **Hydrolysis of Salts** (pH of an acidic/basic salt is simply pH of a weak acid/base.)	**Method 1**: set up ICE table → use $K_a = \frac{K_w}{K_b}$ to solve for $[H^+]$ at equilibrium $\frac{K_w}{K_b} = K_a = \frac{[H^+]^2}{[acid]}$ **Method 2**: acidic salts (from a weak base) $pH = -\log_{10}\sqrt{K_a \times [salt]}$ $K_a = \frac{K_w}{K_b}$	**Method 1**: set up ICE table → use $K_b = \frac{K_w}{K_a}$ to solve for $[OH^-]$ at equilibrium $\frac{K_w}{K_a} = K_b = \frac{[OH^-]^2}{[base]}$ **Method 2**: basic salts (from a weak acid) $pOH = -\log_{10}\sqrt{K_b \times [salt]}$ $K_b = \frac{K_w}{K_a}$
④ **Buffer Solutions**	Ⓐ $pH = pK_a + \log_{10}\frac{[base]}{[acid]}$ (base = buffer species with 1 less H atom; acid = buffer species with 1 more H atom.) This formula applies for **ALL** buffers (acidic or basic). Ⓑ At maximum buffer capacity, [acid] = [base]. Thus, $pH = pK_a$.	

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Worked Example (mixing two strong acid solutions)

Concentrations will be halved.

Calculate the pH of the solution formed when **equal volumes** of dilute sulfuric acid of pH 2.0 and 4.0 are mixed.

The dibasicity of sulfuric acid is not considered here as pH gives direct information about $[H^+]$.

Solution

Assume that 1 dm^3 of solutions are mixed (pH 2 and 4).

For 1 dm^3 at pH 2.0, $n(H^+) = 10^{-2.0}$ mol

For 1 dm^3 at pH 4.0, $n(H^+) = 10^{-4.0}$ mol

total $n(H^+) = 10^{-2.0} + 10^{-4.0} = 0.0101$ mol

total volume of mixture = 2 dm^3

$$[H^+]_{new} = \frac{0.0101}{2} = 0.00505 \text{ mol dm}^{-3}$$

$pH = -\log_{10}[H^+]$

$pH = -\log_{10}(0.00505) = \underline{2.30}$

Worked Example (mixing an acid and a base)

10.0 cm^3 of 0.120 mol dm^{-3} potassium hydroxide is added to 20.0 cm^3 of 0.150 mol dm^{-3} hydrobromic acid at 25 °C.

If a reaction occurs, always find the final amount of species after reaction is complete.

Calculate the pH of the solution formed.

Solution

	H^+	+	OH^-	→	H_2O
Initial amount / mol	3.00×10^{-3}		1.20×10^{-3}		-
Change in amount / mol	-1.20×10^{-3}		-1.20×10^{-3}		-
Final amount / mol	1.80×10^{-3}		0		-

total volume of mixture = 30.0 cm^3 = 0.0300 dm^3

$$[H^+]_{new} = \frac{1.80 \times 10^{-3}}{0.0300} = 0.0600 \text{ mol dm}^{-3}$$

$pH = -\log_{10}[H^+]$

$pH = -\log_{10}(0.0600) = \underline{1.22}$

Recall that mixing always results in a change in concentration.

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Worked Example (weak acid solution)

Given that a 0.500 mol dm^{-3} HNO_2 solution has a K_a value of 7.1×10^{-4}, calculate the pH of the solution and determine the fraction of undissociated HNO_2 molecules at equilibrium.

$$HNO_2(aq) \rightleftharpoons H^+(aq) + NO_2^-(aq)$$

Solution

Let x mol dm^{-3} be the concentration of H^+ at equilibrium.

	$HNO_2(aq)$	$\rightleftharpoons$	$H^+(aq)$	+	$NO_2^-(aq)$
Initial conc. / mol dm^{-3}	0.5		0		
Change in conc. / mol dm^{-3}	$-x$		$+x$		$+x$
Equilibrium conc. / mol dm^{-3}	$0.5 - x$		x		x

$$K_a = \frac{[H^+][NO_2^-]}{[HNO_2]}$$

$$7.1 \times 10^{-4} = \frac{x^2}{0.5 - x}$$

Since the dissociation is weak, we can assume that $0.5 - x \approx 0.5$. Thus,

$$7.1 \times 10^{-4} \approx \frac{x^2}{0.5}$$

$$[H^+] = x \approx 0.0188$$

$$pH = -\lg(0.0188) = \underline{1.72}$$

Alternatively,

Do not put in pK_a value here!

$$[H^+] = \sqrt{K_a \times [\text{acid}]}$$

$$[H^+] = \sqrt{(7.1 \times 10^{-4}) \times 0.5} = 0.0188 \text{ mol dm}^{-3}$$

$$pH = -\log_{10}(0.0188) = \underline{1.72}$$

concentration of undissociated molecules = 0.500 – 0.0188 = 0.481 mol dm^{-3}

fraction of undissociated molecules = $\frac{0.481}{0.500} = \underline{0.962}$

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Salt Hydrolysis

- A **salt** is obtained when the H^+ from the acid is replaced with a metal or ammonium cation.
- When a salt dissolves in water, the resulting solution may be neutral, acidic or basic. Some ions of weak acids or bases will undergo further hydrolysis to give acidic or basic solutions.
- Ions (or conjugates) that come from weak reagents are stronger, and thus can undergo hydrolysis to release some H^+ or OH^- ions, making its solution slightly acidic or basic.

Steps to determine hydrolysis of salt

Step 1. Find the acid and base that reacts together to give the required salt.

- Anions come from acids and cations come from bases.
- Determine the strength of each acid or base.

$CH_3CO_2^-Na^+$	**$NH_4^+Cl^-$**
CH_3CO_2Na is formed from neutralisation between weak acid CH_3CO_2H and strong base NaOH.	NH_4Cl is formed from neutralisation between strong acid HCl and weak base NH_3.

Step 2. Determine the acid-base nature of the ions in the salt.

$CH_3CO_2^-Na^+$	**$NH_4^+Cl^-$**
• $CH_3CO_2^-$ is the conjugate base of weak acid, CH_3CO_2H. • Hence, $CH_3CO_2^-$ is a weak base and hydrolyses to give a basic solution. • Na^+ is a metal ion with low charge density and will not undergo hydrolysis in water.	• NH_4^+ is the conjugate acid of weak base, NH_3 • Hence, NH_4^+ is a weak acid, and hydrolyses to give an acidic solution. • Cl^- is a conjugate base of the **strong acid** HCl, and is **VERY weakly basic**. It will not undergo hydrolysis.

Important Note:

- Metal ions can be weakly acidic if they have high charge density. For e.g., Mg^{2+} or Al^{3+} cations can undergo hydrolysis in water to give a pH of ~6.5 and ~3 respectively.

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- Conjugate bases of strong acids such as Cl^-, Br^-, I^-, SO_4^{2-}, NO_3^- are very weak bases because their K_b values are incredibly small (the K_a of a strong acid is very large ($\approx 10^6$) and hence the K_b of its conjugate base is very small ($< 10^{-14}$) since $K_b = \frac{K_w}{K_a}$)

Step 3. Write down the hydrolysis reaction.

➢ The ion will react with water reversibly to give either H_3O^+ or OH^- to illustrate its acidic or basic behaviour (as determined earlier in step 2).

- (anion from weak base) + $H_2O \rightleftharpoons$ (conjugate) + H_3O^+
- (cation from weak acid) + $H_2O \rightleftharpoons$ (conjugate) + OH^-

$CH_3CO_2^-Na^+$	**$NH_4^+Cl^-$**
$CH_3CO_2^- + H_2O \rightleftharpoons CH_3CO_2H + OH^-$	$NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+$

The Fast Way to Determine if a Salt is Acidic, Basic or Neutral

Step 1: Identify the ions in the salt and ignore these:

- Almost all metal ions (unless the question specifies that the metal ion is acidic and K_a value is provided)
- Conjugate bases of strong acids (Cl^-, Br^-, I^-, NO_3^-, SO_4^{2-})

- H atoms of C–H bonds (e.g., H in $-CH_3$) are **NOT** acidic.
- An acidic H atom is usually a H atom bonded to highly electronegative elements (N or O).

*Step 2: Look at what ion is left.

- positively charged and/or contains an acidic hydrogen atom, the ion is acidic.
- negatively charged, the ion is basic.
- if nothing remains, the salt is neutral.

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***WARNING!**

- Some ions can be acidic and basic (amphoteric) because they contain an acidic hydrogen atom **and** a negative charge.
- These ions usually come from weak polyprotic acids like $H_3\mathbf{A}$ such as $H_2\mathbf{A}^-$ and $H\mathbf{A}^{2-}$.
- For e.g., $H_2PO_4^-$ (an ion of H_3PO_4) can hydrolyse in water to release H^+ ions and OH^- ions:

$$H_2PO_4^-(aq) + H_2O(l) \rightleftharpoons HPO_4^{2-}(aq) + \mathbf{H_3O^+(aq)}$$

$$H_2PO_4^-(aq) + H_2O(l) \rightleftharpoons H_3PO_4(aq) + \mathbf{OH^-(aq)}$$

For e.g.,

	$NaClO_3$	NH_4Cl	Na_2HPO_4
Step 1	disregard Na^+	disregard Cl^-	disregard Na^+
Step 2	remaining ion, ClO_3^- is negatively charged; it is **basic**.	remaining ion, NH_4^+, is positively charged and has an acidic hydrogen atom; it is **acidic**.	remaining ion, HPO_4^{2-} is negatively charged and has an acidic hydrogen atom; it is **acidic and basic**.

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Worked Example

(a) With the aid of a chemical equation, explain why a solution of sodium benzoate is basic.

$C_6H_5CO_2^-$	Na^+
Benzoate ion is the conjugate base of a weak acid, benzoic acid. The benzoate ion undergoes hydrolysis with water to give a weakly basic solution. $C_6H_5CO_2^-(aq) + H_2O(l) \rightleftharpoons C_6H_5CO_2H(aq) + OH^-(aq)$	Na^+ has low charge density and will not undergo hydrolysis.

(b) Calculate the pH of 15 cm^3 of 0.2 mol dm^{-3} of sodium benzoate (K_a of benzoic acid = 6.30×10^{-5} mol dm^{-3}) at 25°C.

Solution

Let x mol dm^{-3} be the concentration of H^+ at equilibrium.

Note that the salt solution of sodium benzoate will be basic:

	$C_6H_5CO_2^-(aq)$	+ $H_2O(l)$	⇌ $C_6H_5CO_2H(aq)$	+ $OH^-(aq)$
Initial conc.	0.2		0	
Change in conc.	$-x$		$+x$	$+x$
Eq. conc.	$0.2 - x$		x	x

$$K_b = \frac{[C_6H_5CO_2H][OH^-]}{[C_6H_5CO_2^-]}$$

$$\frac{10^{-14}}{6.30 \times 10^{-5}} = \frac{x^2}{0.2 - x}$$

Since the dissociation is weak, we can assume that $0.5 - x \approx 0.5$. Thus,

$$1.59 \times 10^{-10} \approx \frac{x^2}{0.2}$$

$$[OH^-] = x \approx 5.63 \times 10^{-6}$$

$$pH = 14 + \lg(5.63 \times 10^{-6}) = \underline{8.75}$$

Alternatively, $pOH = -\lg\sqrt{K_b \times [salt]}$

$$pOH = -\lg\sqrt{\frac{10^{-14}}{6.30 \times 10^{-5}} \times 0.2} = 5.25$$

$$pH = 14 - 5.25 = \underline{8.75}$$

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Buffer Solutions

A **buffer** solution resists pH changes when a small amount of acid or base is added.

- A buffer solution is made up of a weak conjugate acid-base pair.
- A good (or effective) buffer must satisfy the following criteria:
  - amount of conjugate acid-base pair must be large enough relative to the amount of H^+ or OH^- added.
  - $\frac{[\text{base}]}{[\text{acid}]}$ ratio ranges from 0.1 to 10 $\Leftrightarrow$ $pK_a - 1 < pH < pK_a + 1$.

$$pH = pK_a + \log_{10}\frac{[\text{base}]}{[\text{acid}]}$$

There is **no need** to figure out whether the buffer is acidic or basic before applying the buffer formula.
This one formula applies to **ALL** buffers

The base (or acid) is the buffer species that has one fewer (or one more) H^+ ions.

For e.g., in a buffer pair of $H_2PO_4^-$ and HPO_4^{2-}, HPO_4^{2-} is the base and $H_2PO_4^-$ is the acid.

- The **maximum buffering capacity** occurs when the buffer solution is equally good at buffering against both additional acid and base.
  - This occurs when [acid] = [base], or $\frac{[\text{base}]}{[\text{acid}]} = 1$. Equivalently, we have $pH = pK_a$.
- An **acidic** buffer (weak acid and its conjugate base; CH_3COOH and CH_3COONa) maintains the pH of the solution below 7, whereas a **basic** buffer (weak base and its conjugate acid; NH_3 and NH_4Cl) maintains the pH above 7.

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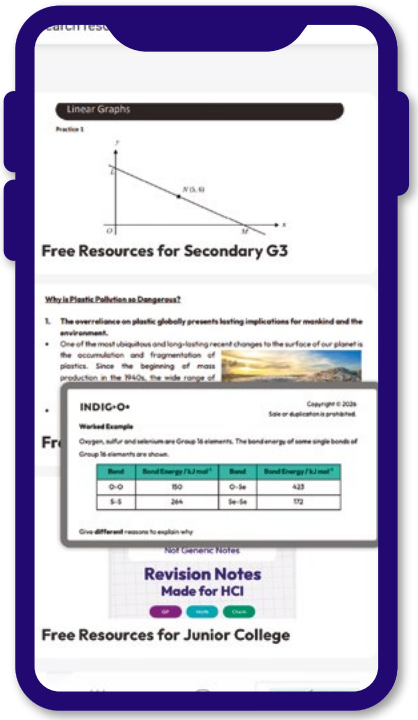

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+65 6634 2208 contact@indigogroup.com.sg indigo.com.sg

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